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An experiment on network density and sequential learning ☆

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ABSTRACT

We conduct a sequential social-learning experiment where subjects each guess a hidden state based on private signals and the guesses of a subset of their predecessors. A network determines the observable predecessors, and we compare subjects' accuracy on sparse and dense networks. Accuracy gains from social learning are twice as large on sparse networks compared to dense networks. Models of naive inference where agents ignore correlation between observations predict this comparative static in network density, while the finding is difficult to reconcile with rational-learning models.

1. Introduction

In many economic situations, people form beliefs based on others' actions. In these settings, agents typically do not observe all members of the society, but only a select subset — namely, their neighbors in an underlying social network. How the structure of this observation network affects learning outcomes is a fundamental question for understanding social learning. While an extensive theoretical literature has explored this question for both naive and rational agents (e.g., Golub and Jackson, 2010; Acemoglu et al., 2011; Golub and Jackson, 2012), much less is known empirically.

Density is one of the most basic properties of a network. How do learning patterns differ between sparse networks, where agents usually observe very few neighbors, and dense networks, where agents generally have abundant social information? On denser networks, agents observe more predecessors (both directly and indirectly), so their actions can incorporate the private signals of more individuals. But whether this leads to more accurate learning ultimately depends on how society aggregates these signals. Predecessors' actions can be correlated by their common neighbors, so this aggregation may be difficult.

In this work, we conduct an experiment to compare social-learning outcomes on sparse and dense networks. We study a sequential social-learning environment where agents on an observation network each guess a hidden state. We find that although later agents have fewer observations on sparser networks, they nevertheless learn substantially better on sparse networks than dense networks.

We place subjects into groups of 40 who act in order. Each group lives on a social network, with randomly-generated links that determine each subject's observations. Each subject has a 25% chance of observing each predecessor in the sparse

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treatment and a 75% chance in the dense treatment (and subjects know these probabilities). A hidden binary state is drawn for each group. On her turn, each subject must guess the state using her private signal and the past guesses of the predecessors she observes. Subjects were paid for accuracy.

Prior to data collection, we pre-registered a measure of long-run learning accuracy: the fraction of the final 8 subjects in the group who correctly guess the state. Comparing this measure on 130 sparse networks versus 130 dense networks, we find that denser networks lead to worse learning accuracy. In dense networks, the average accuracy of the last 8 subjects improves on the autarky benchmark (i.e., the average accuracy if no one can observe others' actions) by 5.7%, but this improvement is 12.6% in sparse networks. Thus, the long-run accuracy gains from social learning are twice as large in the sparse treatment as in the dense treatment (p-value 0.0239).

In addition to its direct implications about the role of network density in social learning, this finding provides indirect evidence supporting models of *naive inference* in which agents neglect the correlations among their social observations (as in Eyster and Rabin, 2010). Motivated by a theoretical result from Dasaratha and He (2020), we compute predictions of the naive model. Later agents exhibit higher accuracy on sparse networks than dense networks in this model, as in our experimental evidence. The basic intuition is that an agent with correlation neglect ends up placing too much weight on the actions of the first few subjects in the same group, as these actions commonly influence many of the agent's predecessors. When the network is denser, this over-weighting is more severe and so naive agents' guesses are less accurate in the long run.

On the other hand, our experimental findings are inconsistent with the rational social-learning model. Acemoglu et al. (2011)'s results imply that rational agents learn asymptotically in environments matching our experimental setup. We adapt their methods to provide lower bounds on the accuracy of rational agents 33 through 40 in the sparse and dense treatments. These bounds imply that rational agents' accuracy cannot improve substantially from the dense-network treatment to the sparse-network treatment — in particular, the rational model does not predict a doubling of accuracy gain.

Our data also show that network density has no statistically significant effect on the *overall* accuracy averaged across all 40 subjects in each group. This is because dense networks increase the accuracy of subjects who move early in the group, even though they lower the accuracy of subjects who move later. This reversal of the accuracy ranking between sparse and dense networks over the course of social learning is another prediction of naive inference.

Finally, to provide additional evidence that learning is worse on denser networks because subjects fail to account for correlation, we conduct a variant of the experiment where subjects observe neighbors who make conditionally independent guesses. The setup is the same as in the main experiment, except the first 32 agents in each group only observe their own private signals, while the final 8 agents randomly observe some of the initial 32 agents. For the latter subjects, average guess accuracy is 68.2% when there is a 25% chance of observing each predecessor and 72.5% when there is a 75% chance of observing each predecessor. The extra observations in dense networks improve guess accuracy when those observations are not correlated by common social information.

1.1. Related literature

Our experimental results add to a growing body of evidence that humans do not properly account for correlations in social-learning settings. Enke and Zimmermann (2017) show that correlation neglect is prevalent even in simple environments where the observed information sources are mechanically correlated. In a field experiment where agents interact repeatedly with the same set of neighbors, Chandrasekhar et al. (2020) find agents fail to account for redundancies.

Most closely related to the present work, the laboratory games in Eyster et al. (2018) and Mueller-Frank and Neri (2015) directly evaluate behavioral assumptions matching ours. Eyster et al. (2018) find that on the complete observation network, many agents choose the best response assuming predecessors are rational while some participants exhibit redundancy neglect. On a more complex network the naive model matches more observations than the rational model, and there is little anti-imitation (which would be required for correct Bayesian inference, as shown in Eyster and Rabin, 2014).[1] Mueller-Frank and Neri (2015) find most observations are consistent with the behavioral assumption we study (which they call quasi-Bayesian updating) in a setting where agents have limited information about the network. These experiments suggest naiveté may be more likely in settings where agents either have a limited knowledge of the true network or the network is known but very complicated. In these settings, the correct Bayesian belief given one's observations can be far from obvious, so agents are more likely to resort to behavioral heuristics.

Unlike this previous work, our experiment tests the comparative statics predictions of naive and rational learning with respect to variations in the learning environment. This allows us to cleanly test redundancy neglect against rational updating. Our approach allows us to focus on long-term learning outcomes—which are the welfare-relevant metrics as we consider changes in the environment—instead of solely on measuring individual behavior.

Several experiments in this literature, including Grimm and Mengel (2018), Chandrasekhar et al. (2020), and Mueller-Frank and Neri (2015), test social learning outcomes under multiple network structures. In these works, changes in network structure largely serve as a robustness check for claims about subject behavior. By considering larger networks and varying density, we show network structures play an important role in learning outcomes and exploit this variation to better understand behavior.

[1] In the complex network, four agents move in each period after observing predecessors from previous periods.

2. Theoretical motivation

2.1. Model

The state of the world $\omega \in \{0, 1\}$ takes one of two possible values with equal probabilities. The set of agents is indexed by $i \in \mathbb{N}$. Agents move in the order of their indices, each acting once.

On her turn, each agent i observes a private signal $s_i \in \mathbb{R}$, as well as the actions of some previous agents. Then, i chooses an action $a_i \in \{0, 1\}$ to maximize the probability that $a_i = \omega$ given her belief about ω.

Private signals (s_i) are i.i.d. and Gaussian conditional on the state of the world. When $\omega = 1$, $s_i \sim \mathcal{N}(1, \sigma^2)$. When $\omega = 0$, $s_i \sim \mathcal{N}(-1, \sigma^2)$. Here $\sigma^2 > 0$ is the conditional variance of the private signal.

In addition to her signal, each agent i observes the action of each predecessor with probability q. These observations are i.i.d. Independence of observations means that whether one agent observes a certain predecessor does not depend on whether a different agent observes the same predecessor. Agents observed by i are called the *neighbors* of i, and the sets of neighbors define a (random) directed network.

We compare two kinds of agents: rational agents and naive agents. Rational agents play the unique perfect Bayesian equilibrium. Naive agents optimize given the following misspecified beliefs:

Assumption 1 *(Naive Inference Assumption). Each agent wrongly believes that each predecessor chooses an action to maximize her expected payoff based solely on her private signal, and not on her observation of other agents.*

Equivalently, naive agents believe that each of their neighbors observe no other agents. Besides the error in Assumption 1, naive agents are otherwise correctly specified and optimize their expected utility given their mistaken beliefs.

Assumption 1 was introduced in a sequential-learning setting where agents observe all predecessors by Eyster and Rabin (2010). Their work refers to this form of inference as "best-response trailing naive inference" (BRTNI).

2.2. Naive and rational behavior

Dasaratha and He (2020) suggest an empirical test for the naive inference assumption: in the context of sequential learning on uniform random networks, does increasing the link-formation probability q cause more inaccurate long-run beliefs? In this paper, we experimentally test this comparative static in networks of 40 agents by comparing learning outcomes in sparse networks (where $q = \frac{1}{4}$) and dense networks (where $q = \frac{3}{4}$).

The naive-learning model and the rational-learning model make competing predictions about this comparative static. The intuition for naive learning comes from Dasaratha and He (2020), which suggests that overweighting due to correlation neglect is more severe on dense networks.[2] We do not expect human subjects to behave exactly according to Assumption 1 — for example, the meta-analysis of Weizsäcker (2010) reports that laboratory subjects in sequential learning games suffer from autarky bias, underweighting their social observations relative to the payoff-maximizing strategy. However, the comparative static prediction of the naive model remains robust even after introducing any fraction of autarkic agents.[3]

The prediction of the naive model is shown in Fig. 1, which plots the probabilities that each of the 40 naive agents will correctly guess the state in sparse and dense networks with $\sigma = 2$. Because naive agents' actions only depend on the number of their predecessors choosing each of the two actions and not the order of these actions, recursively calculating the distributions of actions is computationally feasible (see Appendix A.2 for details). As shown in Fig. 1, early naive agents do worse under $q = \frac{1}{4}$ than $q = \frac{3}{4}$ because there is very little social information, but the comparison quickly reverses as we examine later naive agents.

On the other hand, the rational-learning model predicts that later agents will have either similar or greater accuracy on the dense network compared to the sparse network. Acemoglu et al. (2011)'s results imply that in an environment matching our experimental setup, rational agents will learn the true state in the long-run, regardless of the network density. We can confirm that 40 rational agents are enough to approach this asymptotic learning limit when $q = \frac{3}{4}$. To do this, we compute a lower bound for the probability of correct learning for each agent i in the dense network of our experiment, assuming all agents are rational Bayesians (see Appendix A.1 for details). This lower bound is based on (suboptimal) agent strategies that only depend on own private signals and the action of just one neighbor, as in the neighbor-choice functions in Lobel and Sadler (2015). This exercise shows that the 33$^{\text{rd}}$ rational agent is correct at least 96.8% of the time on dense networks, with the lower bound on accuracy continuing to increase up to the 40$^{\text{th}}$ agent, who is correct at least 97.5% of the time. In addition to suggesting that the asymptotic result of Acemoglu et al. (2011) very likely holds by the 40$^{\text{th}}$ agent, the fact that this lower bound for accuracy on the dense network is so close to perfect learning proves the 40$^{\text{th}}$ rational agent could not

[2] Dasaratha and He (2020) consider agents with a continuous action space, but we implemented a binary action space in the experiment for clarity. We felt it would be easier for subjects to make a binary choice than to accurately report their exact belief.

[3] See the Appendix of a previous version of Dasaratha and He (2020), available at https://arxiv.org/pdf/1703.02105v5.pdf.

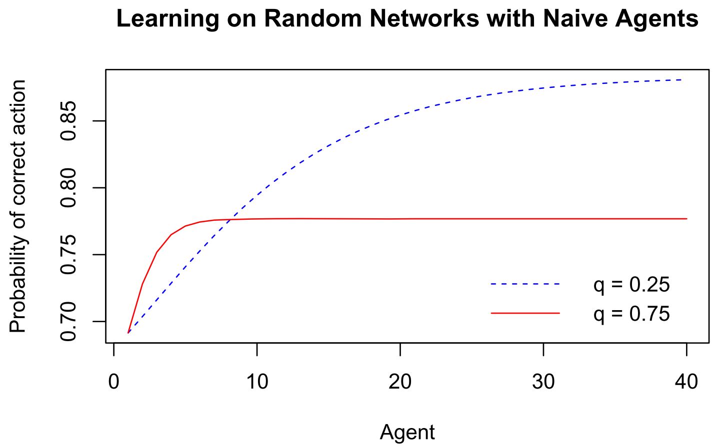

Fig. 1. Learning accuracy on random networks with 40 naive agents, binary actions, and $\sigma^2 = 4$. Dashed blue and solid red curves show the expected accuracy of different agents on networks with link probabilities $q = \frac{1}{4}$ and $q = \frac{3}{4}$, respectively. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

perform substantially better on the sparse network,[4] contrary to the predicted improvement for the 40$^{\text{th}}$ naive agent shown in Fig. 1.

Intuitively one might also expect more connections to also help rational agents in the short- and medium-run as they can adjust for potential redundancies in information. For example, on the complete network with continuous actions, rational agents can back out the private signals of all predecessors by observing their actions, so every agent i does better on the complete network than on any sparser network structure. We note, however, that exact comparative statics of the rational model or variants are not known on random networks.

We experimentally test the competing predictions of the naive and the rational models about how long-run accuracy varies with network density. We thus provide indirect evidence for the naive inference assumption, complementing the direct measurement of behavior in Eyster et al. (2018) and Mueller-Frank and Neri (2015).

Beyond providing another form of evidence, our experiment also contributes to understanding social learning by using the welfare-relevant outcome, namely the long-run accuracy of actions, as the dependent variable. Even if individual behavior tends to match redundancy neglect models in simple or stylized settings, one might worry that the theoretical implications of said models concerning aggregate learning need not hold in practice for complex environments. For a policymaker who can alter the observation network, for instance, experiments using welfare-relevant outcomes as their dependent variables give more explicit guidance as to the consequences of different policies.

3. Experimental design

We conducted our experiment on the online labor platform Amazon Mechanical Turk (MTurk) using Qualtrics survey software.

We pre-registered our experimental protocol and regression specification prior to the start of the experiment in August 2017. Our pre-registration included the target sample size (which was met exactly) and the dependent variable to measure the accuracy of social learning. The pre-registration document can be found on the registry website at `https://aspredicted.org/yp6eq.pdf` and is also included in the Online Appendix.

We recruited 1040 subjects. To be recruited, each subject must correctly answer three comprehension questions (which were scenarios in the game with a dominant choice). An additional 375 MTurk users incorrectly answered one or more comprehension questions and were not allowed to participate in the experiment, based on the pre-registered exclusion criteria. These excluded users were 26.5% of the potential subjects. The experiment was carried out in fall 2017.

In addition to comprehension questions, we restricted to subjects located in the United States who had completed at least 50 previous MTurk tasks with a lifetime approval rate of at least 90%. Subjects were not permitted to participate multiple times in the experiment. There were at most 15 subjects who did not complete all trials, implying a completion rate of at least 98.5%. These non-completers were excluded and replaced by new subjects.

Each *trial* consisted of 40 agents who were asked to each make a binary guess between two *a priori* equally likely states of the world, L (for left) and R (for right). The states were color-coded to make instructions and observations more reader-friendly. Agents are assigned positions in the sequence and move in order. Each MTurk subject participated in 10 trials,

[4] We prove these bounds because we are not aware of a computationally feasible method of calculating or simulating the probability that rational agents are correct. Rahimian et al. (2014) show computing rational actions in another social learning environment is NP-hard.

Table 1
Regression results for the effect of network density on learning outcomes (with robust standard errors).

	FractionCorrect
NetworkDensity	-0.0923 (0.0406)
Constant	0.802 (0.0218)
Observations	260
Adjusted R^2	0.016

all in the same position (depending on when they participated in the experiment). The grouping of subjects into trials was independent across trials. Subjects received \$0.25 for completing the experiment and \$0.25 per correct guess, for a maximum possible payment of \$2.75. Subjects received no feedback about the accuracy of their guesses until they were paid at the conclusion of the experiment. Subjects ordinarily took less than 10 minutes to complete their participation and earned \$2.08 on average, so the incentives were quite large for an MTurk task.

In each trial, every agent received a private signal, which had the Gaussian distribution $\mathcal{N}(-1, 4)$ in state L and the Gaussian distribution $\mathcal{N}(1, 4)$ in state R. These distributions were presented visually in the instructions. Along with the value of their signal, subjects were told the probability of each state conditional on only their private signal.

Each trial was also associated with a density parameter, either $q = \frac{1}{4}$ or $q = \frac{3}{4}$. A random network was generated for each trial by linking each agent with each predecessor with probability q. Each MTurk subject was assigned into either the "sparse" or the "dense" treatment, and then placed into 10 trials either all with $q = \frac{1}{4}$ or all with $q = \frac{3}{4}$. So there were 520 subjects and 130 trials for each treatment. Agents were told the actions of each linked predecessor and the link probability q (but not the full realized network, which could not be presented succinctly).

In each trial, agents viewed their private signal and any social observations and were asked to guess the state. States, signals, and networks were independently drawn across trials. Experimental instructions and an example of a choice screen are shown in the Online Appendix.

4. Results

Let $y_{i,j}$ be the indicator random variable with $y_{i,j} = 1$ if agent i in trial j correctly guesses the state, $y_{i,j} = 0$ otherwise. Define $\tilde{y}_j := \frac{1}{8}\sum_{i=33}^{40} y_{i,j}$ as the fraction of the last 8 agents in trial j who correctly guess the state. We test learning outcomes for the final 8 agents because welfare depends on long-run learning outcomes in large societies and these agents better approximate long-run outcomes. By using only her private signal, an agent can correctly guess the state 69.15% of the time.[5] We call $\tilde{y}_j - 0.6915$ the *gain from social learning* in trial j, as this quantity represents improvement relative to the autarky benchmark.

We find that the average gain from social learning is 8.73 percentage points for the $q = \frac{1}{4}$ treatment and 4.12 percentage points for the $q = \frac{3}{4}$ treatment. Social learning improves accuracy on the sparse networks by twice as much as on the dense networks. To test for statistical significance, we consider the regression

$$\tilde{y}_j = \beta_0 + \beta_1 q_j + \epsilon_j$$

where $q_j \in \{\frac{1}{4}, \frac{3}{4}\}$ is the network density parameter for trial j. Recall that each subject was assigned into ten random trials with the same network density and in the same sequential position. This means for two different trials $j' \neq j''$, the error terms $\epsilon_{j'}$ and $\epsilon_{j''}$ are close to independent since there are likely very few subjects who participated in both trials.

We estimate $\beta_1 = -0.092$ with a p-value of 0.0239 (see Table 1). The results are the same whether we use robust standard errors or not. These findings are consistent with naive updating but not with rational updating, as discussed in Section 2.[6]

This difference in the gains from social learning is not driven by different rates of autarky among the two treatments for the last 8 agents. We say an agent *goes against her signal* if she guesses L when her signal is positive or guesses R when her signal is negative. Within the last 8 rounds, there are 138 instances of agents going against their signals in the $q = \frac{1}{4}$ treatment, which is very close to the 136 instances of the same under the $q = \frac{3}{4}$ treatment. However, when agents go against their signals in the last 8 rounds, they correctly guess the state 81.88% of the time under the $q = \frac{1}{4}$ treatment, but

[5] In fact, subjects in the first position (who have no social observations) correctly use their private signals 93.8% of the time.

[6] We pre-registered average accuracy in the last 8 agents (i.e. last 20% of agents) as the dependent variable for the experiment, but the regression result is robust to other definitions of $\tilde{y}_j$. When $\tilde{y}_j$ encodes average accuracy among the last m agents for any $4 \le m \le 12$ (i.e. between last 10% and last 30% of the agents), the estimate for β_1 remains negative.

only 71.32% of the time under the $q = \frac{3}{4}$ treatment. This shows the observed difference in accuracy is due to social learning being differentially effective on the two network structures.

However, the $q = \frac{3}{4}$ treatment yields better learning outcomes for early agents. For agents 10 through 20, the average guess accuracy is 72.24% under the $q = \frac{1}{4}$ treatment and 73.22% under the $q = \frac{3}{4}$ treatment. As such, if we replace the dependent variable in the pre-registered regression with overall accuracy $\bar{y}_j := \frac{1}{40}\sum_{i=1}^{40} y_{i,j}$, then we do not find a statistically significant estimate for β_1 (p-value of 0.663). This result is consistent with the naive-learning model: according to the predictions of the naive model shown in Fig. 1, early agents are more accurate under $q = \frac{3}{4}$, but later agents are more accurate under $q = \frac{1}{4}$. The point of overtaking happens at a later round in practice than in theory, because our experimental subjects rely more on their private signal than predicted by the naive model,[7] consistent with the meta-analysis of Weizsäcker (2010).

Our experiment was designed to compare long-run learning accuracy on different networks instead of measuring individual behavior. We do not directly test alternate behavioral models for two reasons. First, given the complex signal and network structures, such tests will be very noisy in our data. Second, because the spaces of possible networks and actions have very high dimension, it is computationally infeasible to determine the action that each agent would take under common knowledge of rationality. However, in the next subsection we provide some evidence that our findings are driven by herding under naive inference rather than other behavioral mechanisms.

4.1. *Evidence of naive herding*

In this section, we present three pieces of evidence suggesting that naive herding is the mechanism responsible for the difference in learning accuracy between the two treatments.

(1) Distribution of overall accuracy. Fig. 2 in Appendix B plots the distributions of subjects who correctly guess the state in the $q = \frac{1}{4}$ and $q = \frac{3}{4}$ treatments, across different trials. Compared to the distribution under $q = \frac{1}{4}$, the distribution under $q = \frac{3}{4}$ has more extreme values and a larger standard deviation (11.36 percentage points versus 9.12 percentage points). This is suggestive evidence for naive herding. With denser networks, we simultaneously find more trials where agents do very badly overall (from herding on the wrong state) and more trials where agents do very well overall (from herding on the correct state).

(2) Effect of misleading early signals on the accuracy of later agents. Call a private signal *misleading* if it is positive while the state is L, or if it is negative while the state is R. If naive herding is the mechanism, we would expect misleading signals received by early agents to be more harmful for eventual learning accuracy on denser networks than on sparser networks. On the other hand, a different behavioral mechanism based on the salience of the visible decisions would suggest that early misleading signals are more harmful on sparse networks, since each visible decision is more salient when agents have fewer social observations. To test the naive herding mechanism, we expand our baseline regression to include two additional regressors: the number m_j of the first fifth of agents who receive misleading signals in trial j, and its interaction effect with network density. That is, we estimate

$$\tilde{y}_j = \beta_0 + \beta_1 q_j + \beta_2 m_j + \gamma(q_j m_j) + \epsilon_j.$$

The difference in the marginal effect of a misleading early signal for learning accuracy on the dense network ($q = \frac{3}{4}$) versus on the sparse network ($q = \frac{1}{4}$) is $\frac{1}{2}\gamma$ in the above specification.

As reported in Table 5 in Appendix B, we find $\gamma = 0.05$ with a p-value of 0.0923. This means each misleading signal among the first fifth of agents harms the average accuracy of the last fifth of agents in the same trial by an extra 2.5 percentage points in dense networks compared to sparse networks.

(3) Average uncertainty. Based on simulation evidence, we expect naive agents to exhibit more agreement on denser networks. To test this prediction in the data, we consider for each trial a set of 30 moving windows centered around periods 6, 7, ... 35, with each window spanning 11 consecutive periods. For each trial j and each window w, we compute $r_{j,w} \in \{0, \frac{1}{11}, ..., 1\}$ as the fraction of 11 agents in the window who guessed R, and we let $u_{j,w} := r_{j,w} \cdot (1 - r_{j,w})$ be a measure of uncertainty within the window.[8] In windows where agents exhibit a greater degree of agreement, we will see a lower $u_{j,w}$. Under herding, we expect lower uncertainty on denser networks, as higher density accelerates convergence to a (possibly mistaken) social consensus. We find in the data that the average uncertainty across all trials and all windows is 0.165 on dense networks, and 0.178 on sparse networks. Examining uncertainty in each of the 30 windows w separately, we find average $u_{j,w}$ across trials is lower among dense networks than sparse networks for all but 1 out of 30 windows. Numerically, the naive herding theory predicts lower average $u_{j,w}$ on denser networks in all 30 windows.

[7] The overall frequency of agents going against their signals was 36.8% of the predicted frequency under the naive model.

[8] The value of $u_{j,w}$ would be unchanged if we instead defined $r_{j,w}$ as the fraction of the 11 agents in window w who correctly guessed the state.

5. Neighbors with conditionally independent actions

In our main experiment, we find that denser networks lead to worse social learning by later subjects. We have presented evidence suggesting the mechanism behind this result is that subjects neglect correlation in observed actions. To provide additional evidence for this channel, we now test how network density affects social learning when observed actions are conditionally independent given the state. In this section, we will ask whether more observations help subjects whose neighbors only have private information.

5.1. Experimental design

We also pre-registered the experimental protocol and regression specification for this second experiment, including the dependent variable to measure the accuracy of social learning and the target sample size, prior to the start of the experiment in November 2020. The pre-registration document is included in the Online Appendix and may also be accessed via the registry website at https://aspredicted.org/ag8fr.pdf.

This experiment was also conducted online on MTurk. We recruited 624 subjects, and each subject participated in 10 trials. There were a total of 130 trials. To increase power, each trial included subjects in both sparse and dense treatments. The first 32 subjects in each trial had no neighbors, and chose actions based only on their private signals. Each trial also contained 8 subjects in the sparse treatment and 8 subjects in the dense treatment. Subjects in the sparse treatment observed each of the first 32 subjects in the same trial with probability $q = \frac{1}{4}$ while subjects in the dense treatment observed each of the first 32 subjects with probability $q = \frac{3}{4}$. There were no other observations, so the actions of the observed neighbors are always uncorrelated given the state. In particular, the subjects after the first 32 in each trial never observe each other.

We maintained the state distribution, private signal distribution, and action space from the main experiment. Recruitment and payment were also the same as in the main experiment. The experimental instructions were modified to accurately describe the social information subjects would receive, if any. The first 32 subjects in each trial (like the first subject in each trial in the main experiment) were only asked the one comprehension question that just involves private signals, as the other comprehension questions pertain to subjects who receive social information. Subjects earned an average of \$1.90 per person in this second experiment.

5.2. Results

We find that the average accuracy is 68.2% in the sparse treatment and 72.5% in the dense treatment. When subjects' neighbors only have private information and not social information, having more neighbors improves the accuracy of guesses.

In each trial, we will index the 8 subjects in the sparse treatment as $33, \ldots, 40$ and the 8 subjects in the dense treatment as $41, \ldots, 48$. Let $y_{i,j}$ be the indicator random variable with $y_{i,j} = 1$ if agent i in trial j correctly guesses the state, $y_{i,j} = 0$ otherwise. For each $q \in \left\{\frac{1}{4}, \frac{3}{4}\right\}$, we define $\tilde{y}_j^q$ as the fraction of the 8 subjects in that treatment in trial j who correctly guess the state, so

$$\tilde{y}_j^{\frac{1}{4}} := \frac{1}{8} \sum_{i=33}^{40} y_{i,j} \text{ and } \tilde{y}_j^{\frac{3}{4}} := \frac{1}{8} \sum_{i=41}^{48} y_{i,j}.$$

To test for statistical significance, we consider the regression

$$\tilde{y}_j^q = \beta_0^{uncor} + \beta_1^{uncor} q + \epsilon_{j,q}$$

where $q \in \{\frac{1}{4}, \frac{3}{4}\}$ is the network density parameter. We estimate $\beta_1^{uncor} = 0.087$ with a p-value of 0.0391 (see Table 2).

The difference in average accuracy is again driven by a difference in the value of social information. Recall that a subject goes against her signal if her signal is positive and she chooses L or her signal is negative and she chooses R. Conditional on going against one's own signal, subjects correctly guess the state 53.66% of the time in sparse treatment and 69.39% of the time in dense treatment.

Guesses are in general less accurate in this follow-up experiment than in the main experiment. The subjects in the first 32 positions in each trial had only one comprehension question because their decision problems did not involve any social information. Subjects who did not fully understand the experimental instructions may therefore have been more likely to participate in the experiment in these positions, producing much noisier choices that degrade later subjects' accuracy.[9] There may also be differences in the MTurk subject pool compared to the main experiment, as the second experiment was conducted three years later.

[9] Subjects in the first 32 positions correctly used their private signals only 81.7% of the time.

Table 2
Regression results for the effect of network density on learning outcomes for subjects observing neighbors with only private information (with robust standard errors).

	FractionCorrect
NetworkDensity	0.0865
	(0.0417)
Constant	0.660
	(0.0229)
Observations	260
Adjusted R^2	0.013

The follow-up experiment finds that having more observations improves accuracy when those observations are conditionally uncorrelated. This provides additional evidence that our main result is driven by the failure of subjects to account for correlation in observed actions, rather than by some other mechanism that does not depend on this correlation.

6. Concluding discussion

Our study provides experimental evidence on how the density of the observation network affects people's long-run accuracy in social-learning settings. We find that sparser networks double the accuracy gains from social learning relative to denser networks. While the rational model predicts correct asymptotic social learning with minimal assumptions on the social network, we conjecture that in practice, many structural properties of the network can substantially alter long-run accuracy. Our empirical findings support this conjecture for the case of network density, one of the most canonical network statistics. We leave open the roles of other network structures as promising future work.

We have argued that our experimental results provide evidence for inferential naiveté by analyzing a particular form of behavior (Assumption 1). We conclude by discussing two ways in which the experimental results are potentially consistent with more general models of behavior. First, we have discussed models where all agents are rational or all agents are naive, but a model where only some of the agents suffer from inferential naiveté may be more realistic. Such a model could also generate herding on incorrect beliefs, and this herding may be more likely on denser networks. The exact details depend on how the agents who do not suffer from inferential naiveté reason about others' play. If these agents wrongly believe that others are playing the perfect Bayesian equilibrium strategies, then they will fail to correct the mistakes of naive agents. In this case, early agents' actions can have very disproportionate influence on later agents.

Second, Assumption 1 is a particular form of naive updating that assumes agents entirely neglect correlations in neighbors' actions. Even in homogeneous populations, intermediate forms of naive updating could also generate herding on incorrect beliefs. Our main result suggests inferential naiveté, but does not distinguish between alternate naive models involving some correlation neglect.

Appendix A. Theoretical predictions in the experimental environment

A.1. Bounding the performance of rational agents

Consider 40 rational agents on a random network where each agent is linked to each of her predecessors $\frac{3}{4}$ of the time, i.i.d. across link realizations. Agents know their own neighbors but have no further knowledge about the realization of the random network. The signal structure and payoff structure match the experimental design in Section 3.

We provide a lower bound for the accuracy of agents 33 through 40 in the unique PBE of the social-learning game. We first show that when every player uses the equilibrium strategy, all agents learn at least as well as when everyone uses any *constrained strategy* that chooses an action based on only own private signal and the action of the most recent neighbor. We then exhibit payoffs under one such strategy, which give a lower bound on rational performance.

Fix an arbitrary sequence of constrained strategies (σ_i) where $\sigma_i : S_i \times \{0, 1, \emptyset\} \to \Delta(\{0, 1\})$ is only a function of i's signal s_i and the action of the most recent predecessor that i observes ($\sigma_i(s_i, \emptyset)$ refers to i's play if i does not observe any predecessor). Let a_i denote i's (random) action induced by this sequence of strategies. Let a_i' denote i's (random) action when all agents use the PBE strategy.

Claim 1. For all i, $\mathbb{P}[a_i' = \omega] \geq \mathbb{P}[a_i = \omega]$.

Table 3
Lower bounds on the accuracy of rational agents on dense networks.

agent number	33	34	35	36	37	38	39	40
probability correct	0.9685	0.9695	0.9705	0.9714	0.9723	0.9731	0.9739	0.9746

Proof. The proof is by induction on i and the base case of $i = 1$ is clear. Suppose the claim holds for $i = 1, \ldots, n$. Conditional on agent $n + 1$ observing no predecessors, the claim again holds as in the base case, so we can check the claim conditional on $n + 1$ observing at least one neighbor.

Let j be the most recent neighbor that $n + 1$ observes. Then the rational agent observes s_{n+1}, a'_j for some $j \le n$, and perhaps some other actions while the constrained agent only uses s_{n+1} and a_j in decision-making, where $\mathbb{P}[a'_j = \omega] \ge \mathbb{P}[a_j = \omega]$ by the inductive hypothesis. By garbling the observed action a'_j, the rational agent could construct a random variable with the same joint distribution with ω as the less accurate action a_j. Ignoring information other than s_{n+1} and the garbled a'_j, the rational agent $n + 1$ could therefore follow a strategy that does as well as agent $n + 1$ under the strategy profile (σ_i). So we must have $\mathbb{P}[a'_{n+1} = \omega] \ge \mathbb{P}[a_{n+1} = \omega]$ when everyone uses the PBE strategy. □

We then numerically compute the values for $\mathbb{P}[a_i = \omega]$ under the optimal constrained strategy, which are displayed in Table 3.

A.2. Performance of naive agents

Consider 40 naive agents on a random network where each agent is linked to each of her predecessors with probability q, i.i.d. across link realizations. The signal structure and payoff structure match the experimental design in Section 3.

We will compute the accuracy of each agent by a recursive calculation. Because naive agents' actions do not depend on the order of predecessors, behavior depends only on the number of agents who have played L and the number of agents who have played R as well as the network. We will compute the distribution over the number of agents from the first n who have played L and the number who have played R recursively.

Assume the state is R. Let $P(k, k')$ be the probability that k of the first n agents play L and k' of the first n agents play R. We define $P(k, k') = 0$ if $k < 0$ or $k' < 0$. The posterior log-likelihood of state R for a naive agent observing one action equal to R (and no signal) is

$$\ell = \frac{2}{\sigma^2} \cdot \frac{\mu + \sigma\phi(-\mu/\sigma)}{1 - \Phi(-\mu/\sigma)},$$

where Φ and ϕ are the distribution function and probability density function of a standard Gaussian random variable, respectively.

Then we have the recursive relation

$$\begin{aligned} P(k,k') &= P(k-1,k') \textstyle\sum_{i \le k-1, i' \le k'} B(i,k-1,q) B(i',k',q) \Phi(\frac{\sigma(i-i')\ell - 2\mu\sigma}{2}) + \\ &\quad P(k,k'-1) \textstyle\sum_{i \le k, i' \le k'-1} B(i,k,q) B(i',k'-1,q) [1 - \Phi(\frac{\sigma(i-i')\ell - 2\mu\sigma}{2})], \end{aligned}$$

where $B(i, k, q)$ is the probability a binomial distribution with parameters k and q is equal to i. The first summand gives the probability of agent $k + k'$ choosing L after $k - 1$ predecessors choose L and the remainder choose R, and the second summand gives the probability of agent $k + k'$ choosing R after k predecessors choose L and the remainder choose R. The binomial coefficients correspond to the possible network realizations. Here we use naive inference, which implies that only the number of observed agents choosing each action matters for behavior and not their order.

From these distributions $P(\cdot, \cdot)$ we can compute the probability that agent n chooses the correct action R:

$$\sum_{k=0}^{n} P(k, n-k) \sum_{i \le k, i' \le n-k} B(i,k,q) B(i',n-k,q) [1 - \Phi(\frac{\sigma(i-i')\ell - 2\mu\sigma}{2})].$$

These probabilities, which we compute numerically, are displayed in Table 4 for agents 33 through 40.

Table 4
The accuracy of naive agents on sparse and dense networks.

agent number	33	34	35	36	37	38	39	40
accuracy with $q = 1/4$	0.8773	0.8780	0.8786	0.8792	0.8797	0.8801	0.8805	0.8808
accuracy with $q = 3/4$	0.7768	0.7768	0.7768	0.7768	0.7768	0.7768	0.7768	0.7768

Appendix B. Relegated figures and tables

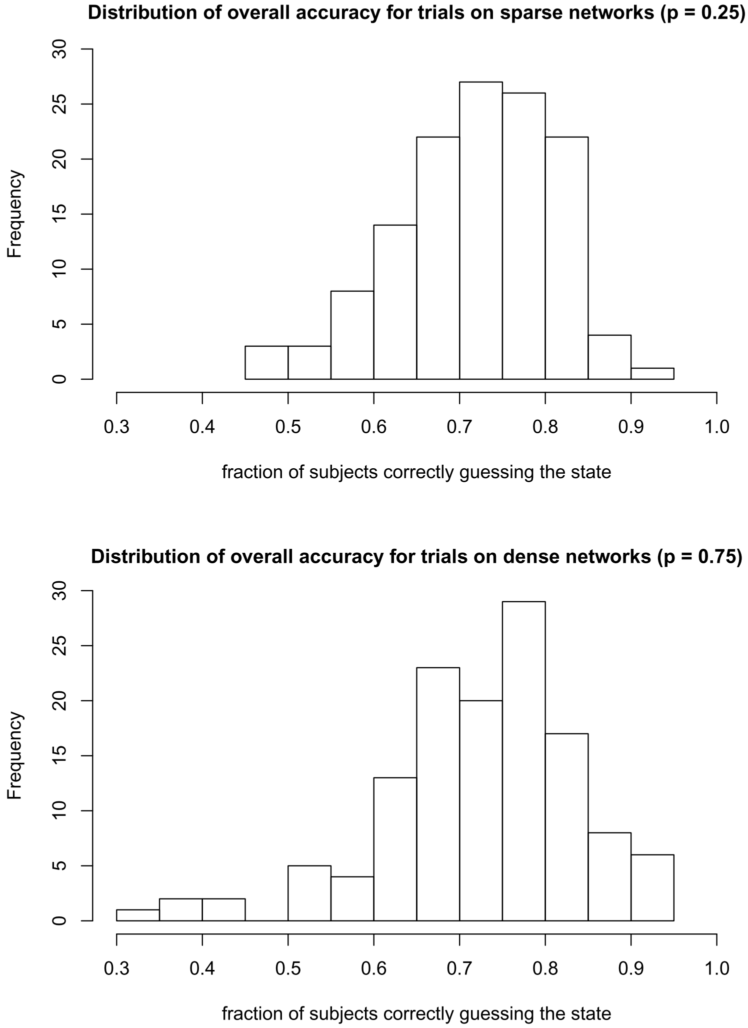

Fig. 2. Histograms of fractions of agents correctly guessing the state.

Table 5
Effect of misleading early signals.

	Dependent variable:
	FractionCorrect
MisleadingEarlySignals	0.014
	(0.017)
NetworkDensity	0.033
	(0.082)
MisleadingEarlySignals×NetworkDensity	−0.050*
	(0.030)
Constant	0.768***
	(0.045)
Observations	260
R^2	0.040
Adjusted R^2	0.029
Residual Std. Error	0.163 (df = 256)
F Statistic	3.566** (df = 3; 256)

Note: *p<0.1; **p<0.05; ***p<0.01.

Appendix C. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.geb.2021.04.004.

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Online Appendix for
"An Experiment on Network Density and Sequential Learning"

Krishna Dasaratha and Kevin He

OA 1 Experimental Instructions

Instructions and an example choice follow. To avoid confusion, the instructions were modified for player 1 in each round to exclude discussion of social observations. A sample experiment can be completed online at https://upenn.co1.qualtrics.com/jfe/form/SV_42dq2J2wHO30zA1

INSTRUCTIONS:

Objective: You will play 10 rounds of a **social-learning game**. At the start of each round, the computer will randomly choose a **direction** for the round, which is either LEFT or RIGHT. Each direction is equally likely to be chosen. A new direction is chosen for each round, which doesn't depend on previous rounds.

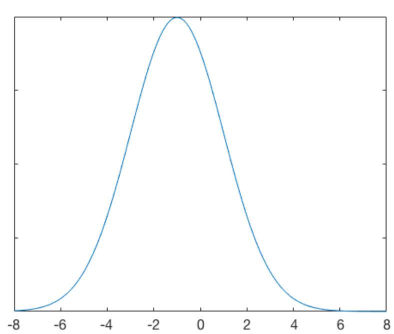
In each round, a number of participants from Amazon Turk will take turns guessing the direction. You will be the **19th** participant to make a guess in each round. You will receive a bonus of $0.25 for each round where you guess the direction correctly.

Private signals: In each round, each participant (including you) will privately receive some information about the direction in the form of a number, which we will call the "**signal**". Different participants will have different signals. When the direction is LEFT, your signal tends to be a more negative number. When the direction is RIGHT, your signal tends to be a more positive number.

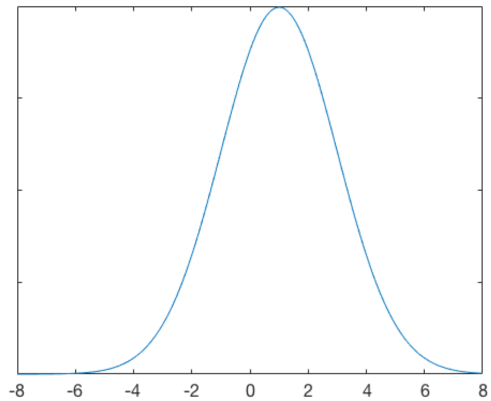
These signals are sometimes imprecise, so it is possible that a few players (maybe including you) could receive negative numbers as their signals even when the direction is RIGHT, for example. However, it is highly likely that most of the players in each round will receive signals that correctly reflect the direction.

Social learning: Since this is a **social**-learning game, in each round you will observe the guesses of some previous participants before you make your own guess. Each participant, including you, observes the guess of each previous participant with 75% chance.

More about the signal: When the direction is LEFT, the distribution of signals is:



When the direction is RIGHT, the distribution of signals is:



In each round, we will also remind you what your own private signal implies about the probabilities of the two directions.

<< >>

This is round 1 (out of 10).

Your signal: -3.3968.
Based on your signal alone, there is 84.53% chance the direction is LEFT, 15.47% chance the direction is RIGHT.

Your observations:
Player 1 guessed RIGHT
Player 3 guessed RIGHT
Player 4 guessed LEFT
Player 5 guessed LEFT
Player 7 guessed RIGHT
Player 8 guessed RIGHT
Player 9 guessed LEFT
Player 10 guessed LEFT
Player 11 guessed LEFT
Player 12 guessed LEFT
Player 13 guessed LEFT
Player 15 guessed LEFT
Player 16 guessed LEFT
Player 17 guessed RIGHT
Player 18 guessed LEFT

What is your guess about the direction this round?

- ○ LEFT
- ○ RIGHT

>>

OA 2 Pre-Registration Documents

AsPredicted
Pre-Registration made easy

Network Structure and Naive Sequential Learning - Experiment (#5250)

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Created: 08/24/2017 06:25 PM (PT)
Public: 09/04/2017 02:10 PM (PT)

1) Have any data been collected for this study already?
No, no data have been collected for this study yet

2) What's the main question being asked or hypothesis being tested in this study?
Subjects in a sequential learning game learn correctly more often with sparser networks, in which most participants observe few neighbors, than with denser networks, in which most participants observe many neighbors.

3) Describe the key dependent variable(s) specifying how they will be measured.
Fraction of the last eight agents in a trial who correctly guess the state.

4) How many and which conditions will participants be assigned to?
Two conditions: each participant observes each predecessor independently with probability p=.25 (the link formation probability) and each participant observes each predecessor independently with probability p=.75.

5) Specify exactly which analyses you will conduct to examine the main question/hypothesis.
Linear regression of the dependent variable on the link formation probability p. The same regression with the dependent variable replaced by the fraction of all agents in a trial who correctly guess the state.

6) Any secondary analyses?
None.

7) How many observations will be collected or what will determine sample size? No need to justify decision, but be precise about exactly how the number will be determined.
Each subject will participate in 10 trials and each trial will contain 40 participants. We intend to enroll 1040 participants (funding permitting).

8) Anything else you would like to pre-register? (e.g., data exclusions, variables collected for exploratory purposes, unusual analyses planned?)
Participants who incorrectly answer comprehension questions will be excluded. Participants who do not complete all trials will be excluded.

ASPREDICTED

Wharton CREDIBILITY LAB
UNIVERSITY of PENNSYLVANIA

Network Density and Sequential Learning – When Neighbors Have No Neighbors (#51451)

Created: 11/05/2020 04:38 PM (PT)
Public: 11/11/2020 05:23 PM (PT)

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1) Have any data been collected for this study already?
No, no data have been collected for this study yet.

2) What's the main question being asked or hypothesis being tested in this study?
Previously, we found that subjects in a sequential learning game learn correctly more often in sparser networks, where most participants observe few neighbors, than in denser networks, where most participants observe many neighbors. Now, to distinguish between the naive inference mechanism from some other possible mechanisms as an explanation for this result, we test whether the same pattern still holds across network density if subjects observe social neighbors who had no social neighbors themselves.

3) Describe the key dependent variable(s) specifying how they will be measured.
Fraction of the last eight agents in a trial who correctly guess the state.

4) How many and which conditions will participants be assigned to?
We will create 130 pairs of matched trials. For each pair of matched trials, we generate a binary state of the world and a profile of 40 signals for 40 participants. The same state and signal profile are used for both trials in the pair. One trial in the pair will have a sparse network, and the other a dense network. For the sparse trial, for each 33 <= i <= 40 and 1 <= j <= 32, we generate a link from i to j with probability 25%. For the dense trial, each such link is generated with probability 75%. In particular, the first 32 participants in each trial have no social neighbors, and the final 8 participants in each trial never observe each other as social neighbors. A participant will either be in an early position (32 or earlier), a late position in a sparse trial, or a late position in a dense trial. A participant in an early position is assigned to 10 random pairs of trials in that position. In each pair of trials, the participant is shown their private signal and asked to make a guess about the state. Each of these guesses is recorded for both trials in the pair. A participant in a late position will be assigned to 10 random trials with the same network density in that position. They observe their private signal and the actions of their network neighbors, then make a guess about the state.

5) Specify exactly which analyses you will conduct to examine the main question/hypothesis.
Linear regression of the dependent variable on the link formation probability p.

6) Describe exactly how outliers will be defined and handled, and your precise rule(s) for excluding observations.
Participants who incorrectly answer comprehension questions will be excluded. Participants who do not complete all trials will be excluded.

7) How many observations will be collected or what will determine sample size? No need to justify decision, but be precise about exactly how the number will be determined.
We will run 130 pairs of trials. As explained above, each of the first 32 pairs of guesses in each pair of trials is made by the same participant. Each participant will be asked to make 10 guesses. Therefore, the total number of participants is (130 x 32 + 130 x 8 x 2) / 10 = 624.

8) Anything else you would like to pre-register? (e.g., secondary analyses, variables collected for exploratory purposes, unusual analyses planned?)
None.