Dynamic Information Design with Diminishing Sensitivity Over News*

Jetlir Duraj† Kevin He‡

First version: July 1, 2019
This version: October 12, 2019

Abstract

A benevolent sender communicates non-instrumental information over time to a Bayesian receiver who experiences gain-loss utility over changes in beliefs (“news utility”). We show how to compute the optimal dynamic information structure for arbitrary news-utility functions. With diminishing sensitivity over the magnitude of news, one-shot resolution of uncertainty is strictly suboptimal under commonly used functional forms. Information structures that deliver bad news gradually are never optimal. We identify additional conditions that imply the sender optimally releases good news in small pieces but bad news in one clump. When the sender lacks commitment power, diminishing sensitivity leads to a credibility problem for good-news messages. Without loss aversion, the babbling equilibrium is essentially unique. More loss-averse receivers may enjoy higher equilibrium news-utility, contrary to the commitment case. We discuss applications to media competition and game shows.

*We thank Drew Fudenberg, Jerry Green, Jonathan Libgober, Pietro Ortoleva, Matthew Rabin, Collin Raymond, the MIT information design reading group, and our seminar participants for insightful comments. We also benefited from conversations with Krishna Dasaratha, Ben Enke, Simone Galperti, David Hagmann, Marina Halac, Johannes Hörner, David Laibson, Shengwu Li, Elliot Lipnowski, Gautam Rao, and Tomasz Strzalecki at an early stage of the project.

†Harvard University. Email: duraj@g.harvard.edu
‡California Institute of Technology and University of Pennsylvania. Email: hesichao@gmail.com
1 Introduction

When people give others news, they are often mindful of the information’s psychological impact. For example, this consideration affects the way CEOs announce earnings forecasts to shareholders and organization leaders update their teams about recent developments. While the instrumental value of information also plays a significant role, we analyze the under-studied problem of how the audience’s psychological reaction to good and bad news shapes the dynamic communication of information. This problem is even more relevant in situations like designing game shows and other entertainment content, where the audience experiences positive and negative reactions over time to news and developments that have no bearing on their personal decision-making.

We consider an informed, benevolent sender communicating non-instrumental information to a receiver who experiences gain-loss utility over changes in beliefs (“news utility”). The state of the world, privately known to the sender, determines the receiver’s consumption at some future date. The sender communicates this state over multiple periods as to maximize the receiver’s expected welfare, knowing that the receiver derives utility based on the nature and the magnitude of news each period — good news elates and bad news disappoints. The receiver will exogenously learn the true state just before future consumption.

We focus on how the receiver’s diminishing sensitivity over news affects the optimal design of information structures. Kahneman and Tversky (1979)’s original formulation of prospect theory envisioned a gain-loss utility component based on deviations from a reference point, where larger deviations carry smaller marginal effects. This idea of diminishing sensitivity is referenced in virtually all subsequent work on reference-dependent preferences, including Köszegi and Rabin (2009), who first introduced a model of news utility. In almost all cases, however, researchers then specialize for simplicity to a two-part linear gain-loss utility function that allows for loss aversion but not diminishing sensitivity. Four decades since Kahneman and Tversky (1979)’s publication, O’Donoghue and Sprenger (2018)’s review of the ensuing literature summarizes the situation as follows:

“Most applications of reference-dependent preferences focus entirely on loss aversion, and ignore the possibility of diminishing sensitivity [...] The literature still needs to develop a better sense of when diminishing sensitivity is important.”

We argue that diminishing sensitivity over the magnitude of news generates novel predic-
tions for information design. As Kőszegi and Rabin (2009) point out, the two-part linear news-utility model makes the stark prediction that people prefer resolving all uncertainty in one period (“one-shot resolution”) over any other dynamic information structure. We show that diminishing sensitivity over news complicates the sender’s problem and leads to a more nuanced optimal information structure. In particular, one-shot resolution is strictly suboptimal for a class of news-utility functions exhibiting diminishing sensitivity. This class includes the commonly used power-function specification. It also includes a tractable quadratic specification, whenever diminishing sensitivity is sufficiently strong relative to the degree of loss aversion. We further identify conditions that imply the optimal information structure treats good news and bad news asymmetrically, disclosing good news gradually but bad news all at once. The direction of this optimal skewness is a central implication of diminishing sensitivity: the “opposite” kind of information structure that divulges all good news at once but doles out bad news in small portions is never optimal. In fact, this kind of information structure is even worse than one-shot resolution.

In our model, the receiver knows the sender’s strategy and formulates Bayesian beliefs. This framework leads to cross-state constraints on the sender’s problem. In view of diminishing sensitivity, one might conjecture that the sender should concentrate all bad news in period 1 if the state is bad, and deliver equally-sized pieces of good news in periods 1, 2, 3,... if the state is good. But these belief paths are infeasible, since a Bayesian audience who knows this strategy and does not receive bad news in period 1 will conclusively infer that the state is good. The receiver should not judge subsequent communication from the sender as further good news or derive positive news utility from them. We show that the sender can nevertheless implement a “gradual good news, one-shot bad news” information structure for a Bayesian receiver by sending a conclusive bad-news signal in a random period when the state is bad. In the optimal information structure, conditional on the good state, the receiver may get different amounts of good news in different periods, even though his news-utility function is time-invariant and the sender knows the state from the start.

Another implication of diminishing sensitivity is that people with opposite consumption rankings over states may exhibit opposite informational preferences. In a world with two possible states, A and B, suppose state A realizes if and only if a series of intermediate events

\[\text{In the language of information design, these conjectured belief paths violate Bayesian plausibility, as they cannot arise from the Bayesian updating of a given prior.}\]
all occur successfully. We show that agents who prefer the consumption they get in state $A$ will choose to observe the intermediate events resolve in real-time (gradual information), while agents who prefer the consumption they get in state $B$ will choose to only learn the final state (one-shot information). This prediction distinguishes the news-utility model with diminishing sensitivity from other models of non-instrumental information preference. The result also rationalizes a “sudden death” format often found in game shows, where the contestant must overcome every challenge in a sequence to win the grand prize (as opposed to the grand prize being contingent on beating at least one of several challenges.)

When the sender lacks commitment power, information structures featuring gradual good news encounter a credibility problem. In the bad state, the sender may strictly prefer to lie and convey a positive message intended for the good state. This temptation exists despite the fact that the sender is far-sighted and maximizes the receiver’s total news utility over time. The intuition is that the receiver will inevitably feel disappointed upon learning the truth in the future, so his marginal utility of (unwarranted) good news today is larger than his marginal disutility of heightened disappointment in the future, thanks to diminishing sensitivity. This perverse incentive to provide false hope in the bad state may preclude all meaningful communication in all states. We show that if the receiver has diminishing sensitivity but not loss aversion (or has low loss aversion), then every equilibrium is payoff-equivalent to the babbling equilibrium. High enough loss aversion, however, can restore the equilibrium credibility of good-news messages by increasing the future disappointment costs associated with inducing false hope today. As a consequence, receivers with higher loss aversion may enjoy higher equilibrium payoffs, which would never happen if the sender had commitment power.

Finally, we characterize the entire family of equilibria featuring gradual good news and study how quickly the receiver learns the state. For a class of news-utility functions that include the square-root and quadratic specifications mentioned before, the sender conveys progressively larger pieces of good news over time, so the receiver’s equilibrium belief grows at an increasing rate in the good state. This puts a uniform bound on the number of periods of informative communication across all time horizons and all equilibria.

The rest of the paper is organized as follows. The remainder of Section 1 reviews related literature. Section 2 defines the sender’s problem under the commitment assumption and introduces our model of news utility. Section 3 studies the optimal information structure and
the relationship between consumption preferences and informational preferences. Section 4 focuses on the cheap-talk model when the sender lacks commitment power. Section 5 looks at a variant of the model without a deterministic horizon. Section 6 discusses other models of preference over non-instrumental information. Section 7 concludes.

1.1 Related Literature

Since Kőszegi and Rabin (2009), several other authors have analyzed the implications of news utility in such varied settings as asset pricing (Pagel, 2016), life-cycle consumption (Pagel, 2017), portfolio choice (Pagel, 2018), and mechanism design (Duraj, 2018). These papers focus on Bayesian agents with two-part linear gain-loss utilities and do not study the role of diminishing sensitivity to news.

We are not aware of other work that focuses on how diminishing sensitivity matters for information design with news utility. In fact, very few papers deal with diminishing sensitivity in any kind of reference-dependent preference. One exception is Bowman, Minehart, and Rabin (1999), who study a consumption-based reference-dependent model with diminishing sensitivity. A critical difference is that their reference points are based on past habits, not rational expectations. In their environment, a consumer who knows their future income optimally concentrates all consumption losses in the first period if income will be low, but spreads out consumption gains across multiple periods if income will be high. As discussed before, the analog of this strategy cannot be implemented in our setting since the receiver derives news utility from changes in rational Bayesian beliefs.

Our model of diminishing sensitivity over the magnitude of news shares the same psychological motivation as Kahneman and Tversky (1979), who base their theory of human responses to monetary gains and losses on human responses to changes in physical attributes like temperature or brightness:

“Many sensory and perceptual dimensions share the property that the psychological response is a concave function of the magnitude of physical change. For example, it is easier to discriminate between a change of 3° and a change of 6° in room temperature, than it is to discriminate between a change of 13° and a change of 16°.”

We are not aware of any empirical work designed to measure diminishing sensitivity over
news, but will highlight some testable predictions of the model later on.

While some of our results apply to Köszegi and Rabin (2009)’s model of news utility or to a more general class of such models (e.g., Proposition 1, Proposition 2, Proposition 3, Corollary A.1), we mostly focus on the simplest model of news utility where the agent derives gain-loss utility from changes in expected future consumption utility. This mean-based model lets us concentrate on the implications of diminishing sensitivity, but differs from Köszegi and Rabin (2009)’s model where agents make a percentile-by-percentile comparison between old and new beliefs. Fully characterizing the optimal information structure using this percentile-based model is out of reach for us, but our numerical simulations in Appendix B.2 suggest the answers would be very similar.

Parallel to the recent literature on the applications of news utility discussed above, Dillenberger and Raymond (2018) axiomatize a general class of additive belief-based preferences in the domain of two-stage lotteries by suitably weakening the independence axiom of expected utility. In the case of \( T = 2 \), our news-utility model belongs to the class they characterize. Under this specialization, our work may be thought of as studying the information design problem, with and without commitment, using some of Dillenberger and Raymond (2018)’s additive belief-based preferences. Dillenberger and Raymond (2018) also provide high-level conditions for additive belief-based preferences to exhibit preference for one-shot resolution. We are able to find more interpretable and easy-to-verify conditions for the sub-optimality of one-shot resolution, working with a specific sub-class of their preferences.

In general, papers on belief-based utility have highlighted two sources of felicity: levels of belief about future consumption utility ("anticipatory utility," e.g., Köszegi (2006); Eliaz and Spiegler (2006); Schweizer and Szech (2018)) and changes in belief about future consumption utility ("news utility"). News utility is a function of both the prior belief and the posterior belief, while a given posterior belief brings the same anticipatory utility for all priors (Eliaz and Spiegler, 2006). As we discuss in Section 6, the rich dynamics of the optimal information structure are a unique feature of the news-utility model (with diminishing sensitivity).

Brunnermeier and Parker (2005) and Macera (2014) study the optimal design of beliefs for agents with belief-based utilities that differ from the news-utility setup we consider. Another important distinction is that we focus on the design of information: changes in the receiver’s belief derive from Bayesian updating an exogenous prior, using the information conveyed by the sender. Macera (2014) considers a non-Bayesian agent who freely chooses
a path of beliefs, while knowing the actual state of the world. Brunnermeier and Parker (2005) study the “opposite” problem to ours, where the agent freely chooses a prior belief (over the sequence of state realizations) at the start of the game, then updates belief about future states through an exogenously given information structure.

Our emphasis on information is shared by Ely, Frankel, and Kamenica (2015), who study dynamic information design with a Bayesian receiver who derives utility from suspense or surprise. In contrast to these authors who propose and study an original utility function over belief paths where larger belief movements always bring greater felicity, we consider a gain-loss utility function over changes in beliefs. Because our states are associated with different consumption consequences, changes in beliefs may increase or decrease the receiver’s utility depending on whether the news is good or bad. While one-shot resolution is suboptimal in both Ely, Frankel, and Kamenica (2015)’s problem and our problem (under some conditions), the optimal information structure differs. The optimal information structure in our problem is asymmetric, a key implication of diminishing sensitivity. Another difference is that information structures featuring gradual bad news, one-shot good news are worse than one-shot resolution in our problem, while one-shot resolution is the worst possible information structure in Ely, Frankel, and Kamenica (2015)’s problem.

Also within the dynamic information design literature but without behavioral preferences, Li and Norman (2018) and Wu (2018) consider a group of senders moving sequentially to persuade a single receiver. The receiver takes an action after observing all signals. This action, together with the true state of the world, determines the payoffs of every player. While these authors study a dynamic environment, only the final belief of the receiver at the end of the last period matters for the players’ payoffs. Indeed, every equilibrium in their setting can be converted into a payoff-equivalent “one-step” equilibrium where the first sender sends the joint signal implied by the old equilibrium, while all subsequent senders babble uninformatively. In our setting, the distribution of the receiver’s final belief at the end of the last period is already pinned down by the prior belief at the start of the first period. Yet, different sequences of interim beliefs cause the receiver to experience different amounts of total news utility. The stochastic process of these interim beliefs constitutes the object of design. We provide a general procedure for computing the optimal dynamic information structure in this new setting.

Lipnowski and Mathevet (2018) study a static model of information design with a psycho-
logical receiver whose welfare depends directly on posterior belief. They discuss an application to a mean-based news-utility model without diminishing sensitivity in their Appendix A, finding that either one-shot resolution or no information is optimal. We focus on the implications of diminishing sensitivity and derive specific characterizations of the optimal information structure. Our work also differs in that we study a dynamic problem, examine equilibria without commitment, and discuss how the rate of releasing good news changes over time.

2 Model

2.1 Timing of Events

We consider a discrete-time model with periods $0, 1, 2, ..., T$, where $T \geq 2$. There are two players, the sender (“she”) and the receiver (“he”). There is a finite state space $\Theta$ with $|\Theta| = K \geq 2$. In state $\theta$, the receiver will consume $c_\theta$ in period $T$, deriving from it consumption utility $v(c_\theta)$ where $v$ is strictly increasing. Assume that $c_{\theta'} \neq c_{\theta''}$ when $\theta' \neq \theta''$. We may normalize without loss $\min_{\theta \in \Theta}[v(c_\theta)] = 0$, $\max_{\theta \in \Theta}[v(c_\theta)] = 1$. There is no consumption in other periods and neither player can affect period $T$’s consumption.

The players share a common prior belief $\pi_0 \in \Delta(\Theta)$ about the state, where $\pi_0(\theta) > 0$ for all $\theta \in \Theta$. In period 0, the sender commits to a finite message space $M$ and a strategy $\sigma = (\sigma_t)_{t=1}^{T-1}$, where $\sigma_t(\cdot \mid h^{t-1}, \theta) \in \Delta(M)$ is a distribution over messages in period $t$ that depends on the public history $h^{t-1} \in H^{t-1} := (M)^{t-1}$ of messages sent so far, as well as the true state $\theta$. The sender can commit to any information structure $(M, \sigma)$, which becomes common knowledge between the players. At the start of period 1, the sender privately observes the state’s realization, then sends a message in each of the periods $1, 2, ..., T - 1$ according to the strategy $\sigma$. (Section 4 studies a cheap talk model where the sender lacks commitment power.) Information about $\theta$ is non-instrumental in that it does not help the receiver make better decisions, but it can change his belief about future welfare.

At the end of period $t$ for $1 \leq t \leq T - 1$, the receiver forms the Bayesian posterior belief $\pi_t$ about the state after the on-path history $h^t \in H^t$ of $t$ messages. This belief is rational and calculated with the knowledge of the information structure $(M, \sigma)$. In period $T$, the receiver exogenously and perfectly learns the true state $\theta$, consumes $c_\theta$, and the game ends. (Section
5 considers a random-horizon model where the termination date is random and unknown to both parties.)

Since the receiver is Bayesian, the sender faces cross-state constraints in choosing paths of beliefs. For example, if the sender wishes to use some message \( m \in M \) to convey positive but inconclusive news in the first period when the state is good, then the same message must also be sent with positive probability when the state is bad – otherwise, receiving this information in the first period would amount to conclusive evidence of the good state. As we later show, these cross-state constraints imply distortions from perfect “consumption smoothing” of good news.

When \( K = 2 \), we label two states as \( \text{Good} \) and \( \text{Bad}, \) \( \Theta = \{G, B\} \), so that \( v(c_G) = 1 \), \( v(c_B) = 0 \). We also abuse the notation \( \pi_t \) to mean \( \pi_t(G) \) in the case of binary states.

In this model, the sender has perfect information about the receiver’s future consumption level once she observes the state. Appendix B discusses an extension where the sender’s information is imperfect, so that there is residual uncertainty about the receiver’s consumption even conditional on the state (i.e., conditional on the sender’s private information).

### 2.2 News Utility

The receiver derives utility based on changes in his belief about the final period’s consumption. Specifically, he has a continuous news-utility function \( N : \Delta(\Theta) \times \Delta(\Theta) \to \mathbb{R} \), mapping his pair of new and old beliefs about the state into a real-valued felicity.\(^2\) He receives utility \( N(\pi_t \mid \pi_{t-1}) \) at the end of period \( 1 \leq t \leq T \). Utility flow is undiscounted and the receiver has the same \( N \) in all periods. The sender maximizes the total expected welfare of the receiver, which is the sum of the news utilities in different periods and the final consumption utility, \( \sum_{t=1}^{T} N(\pi_t \mid \pi_{t-1}) + v(c) \). We assume for every \( \pi \in \Delta(\Theta) \), both \( N(\cdot \mid \pi) \) and \( N(\pi \mid \cdot) \) are continuously differentiable except possibly at \( \pi \).

For many of our results, we study a mean-based news-utility model. Kőszegi and Rabin (2009) mention this model, but mostly consider a decision-maker who makes a percentile-by-percentile comparison between his old and new beliefs. We use the mean-based model to focus on the implications of diminishing sensitivity in the simplest setup. The agent applies a gain-loss utility function, \( \mu : [-1, 1] \to \mathbb{R} \), to changes in expected consumption utility for

---

\(^2\)Since different states lead to different levels of consumption, beliefs over states induce beliefs over consumption.
period $T$. That is, $N(\pi_t | \pi_{t-1}) = \mu(\sum_{\theta \in \Theta}(\pi_t(\theta) - \pi_{t-1}(\theta)) \cdot v(c_0))$. Throughout we assume $\mu$ is continuous, strictly increasing, twice differentiable except possibly at 0, and $\mu(0) = 0$. We impose further assumptions on $\mu$ to reflect diminishing sensitivity and loss aversion.

**Definition 1.** Say $\mu$ satisfies *diminishing sensitivity* if $\mu''(x) < 0$ and $\mu''(-x) > 0$ for all $x > 0$. Say $\mu$ satisfies *(weak) loss aversion* if $-\mu(-x) \geq \mu(x)$ for all $x > 0$. There is *strict loss aversion* if $-\mu(-x) > \mu(x)$ for all $x > 0$.

We now discuss two important functional forms of $\mu$. In Appendix B.2, we compare the optimal information structures for this model and for Kőszegi and Rabin (2009)'s percentile-based model, a class of news-utility functions that do not admit a mean-based representation.

### 2.2.1 Quadratic News Utility

The quadratic news-utility function $\mu : [-1, 1] \to \mathbb{R}$ is given by

$$
\mu(x) = \begin{cases} 
\alpha_p x - \beta_p x^2 & x \geq 0 \\
\alpha_n x + \beta_n x^2 & x < 0 
\end{cases}
$$

with $\alpha_p, \beta_p, \alpha_n, \beta_n > 0$. So we have

$$
\mu'(x) = \begin{cases} 
\alpha_p - 2\beta_p x & x \geq 0 \\
\alpha_n + 2\beta_n x & x < 0 
\end{cases}, \quad \mu''(x) = \begin{cases} 
-2\beta_p & x \geq 0 \\
2\beta_n & x < 0 
\end{cases}.
$$

The parameters $\alpha_p, \alpha_n$ control the extent of loss aversion near 0, while $\beta_p, \beta_n$ determine the amount of curvature — i.e., the second derivative of $\mu$. We only consider quadratic news-utility functions that satisfy the following parametric restrictions.

1. **Monotonicity:** $\alpha_p \geq 2\beta_p$ and $\alpha_n \geq 2\beta_n$. Monotonicity condition holds if and only if $\mu'(x) \geq 0$ for all $x \in [-1, 1]$.

2. **Loss aversion:** $\alpha_n - \alpha_p \geq (\beta_n - \beta_p)z$ for all $z \in [0, 1]$. This condition is equivalent to loss aversion from Definition 1 for this class of news-utility functions.

A family of quadratic news-utility functions that satisfy these two restrictions can be constructed by choosing any $\alpha \geq 2\beta > 0$ and $\lambda \geq 1$, then set $\alpha_p = \alpha$, $\alpha_n = \lambda \alpha$, $\beta_p = \beta$, $\beta_n = \lambda \beta$. Figure 1 plots some of these news-utility functions for different values of $\alpha, \beta,$ and $\lambda$. 


2.2.2 Power-Function News Utility

The power-function news-utility $\mu : [-1, 1] \to \mathbb{R}$ is given by

$$\mu(x) = \begin{cases} 
    x^\alpha & x \geq 0 \\
-\lambda|x|^\beta & x < 0 
\end{cases}$$

with $0 < \alpha, \beta < 1$ and $\lambda \geq 1$. Parameters $\alpha, \beta$ determine the degree of diminishing sensitivity to good news and bad news, while $\lambda$ controls the extent of loss aversion. This class of functions nests the square-root case when $\alpha = \beta = 0.5$ and is the only class of gain-loss functions to appear in Tversky and Kahneman (1992).

3 Optimal Information Structure

In this section, we characterize the optimal information structure that solves the sender’s problem. We provide a general inductive procedure to maximize total expected news utility and find an information structure with $K$ messages that achieves this maximum. We show that information structures featuring gradual bad news, one-shot good news are strictly
worse than one-shot resolution, then identify sufficient conditions that imply the optimal
information structure features gradual good news, one-shot bad news. We illustrate these
conditions with the quadratic news-utility specification, finding that the conditions hold
whenever diminishing sensitivity is sufficiently strong relative to loss aversion.

We conclude this section by highlighting that agents with opposite consumption pref-
erences over two states of the world can exhibit opposite informational preferences when
choosing between one-shot resolution and gradual resolution of uncertainty. This endoge-
nous diversity of information preferences distinguishes news utility with diminishing sensi-
tivity from other models of preference over non-instrumental information in the literature.

3.1 A General Backwards-Induction Procedure

For $f : \Delta(\Theta) \rightarrow \mathbb{R}$, let $\text{cav} f$ be the concavification of $f$ — that is, the smallest concave func-
tion that dominates $f$ pointwise. Concavification plays a key role in solving this information
design problem, just as in Kamenica and Gentzkow (2011) and Aumann and Maschler (1995).

For $\pi_{T-2}, \pi_{T-1} \in \Delta(\Theta)$ two beliefs about the state, let $U_{T-1}(\pi_{T-1} \mid \pi_{T-2})$ be the sum of
the receiver’s expected news utilities in periods $T - 1$ and $T$, if he enters period $T - 1$ with
belief $\pi_{T-2}$ and updates it to $\pi_{T-1}$. More precisely,

$$U_{T-1}(\pi_{T-1} \mid \pi_{T-2}) := N(\pi_{T-1} \mid \pi_{T-2}) + \sum_{\theta \in \Theta} \pi_{T-1}(\theta) \cdot N(1_{\theta} \mid \pi_{T-1}),$$

where $1_{\theta}$ is the degenerate belief putting probability 1 on the state $\theta$. Note that by the
martingale property of beliefs, if the receiver holds belief $\pi_{T-1}$ at the end of period $T - 1$,
then state $\theta$ must then realize in period $T$ with probability $\pi_{T-1}(\theta)$.

Let $U_{T-1}^{*}(\pi_{T-2}) := (\text{cav} U_{T-1}(\cdot \mid \pi_{T-2}))(\pi_{T-2})$. As we will show in the proof of Proposition
1, $U_{T-1}^{*}(\pi_{T-2})$ is the value function of the sender when the receiver enters period $T - 1$ with
belief $\pi_{T-2}$. It is calculated by evaluating the concavified version of $x \mapsto U_{T-1}(x \mid \pi_{T-2})$
at the point $x = \pi_{T-2}$. By Carathéodory’s theorem, there exist weights $w^1, ..., w^K \geq 0$,
beliefs $q^1, ..., q^K \in \Delta(\Theta)$, with $\sum_{k=1}^{K} w^k = 1$, $\sum_{k=1}^{K} w^k q^k = \pi_{T-2}$, such that $U_{T-1}^{*}(x) = \sum_{k=1}^{K} w^k U_{T-1}(q^k \mid x)$. When the receiver enters period $T - 1$ with belief $\pi_{T-2}$, the sender
maximizes his expected payoff using a signaling strategy $\sigma_{T-1}$ that generates a distribution
of posteriors supported on $(q^1, ..., q^K)$ with probabilities $(w^1, ..., w^K)$.
Continuing inductively, using the value function $U^*_{t+1}(x)$ for $t \geq 1$, we may define:

$$U_t(\pi_t \mid \pi_{t-1}) := N(\pi_t \mid \pi_{t-1}) + U^*_{t+1}(\pi_t),$$

which leads to the period $t$ value function $U^*_t(x) := (\text{cav}U_t(\cdot \mid x))(x)$. The maximum expected news utility across all information structures is $U^*_1(\pi_0)$.

Proposition 1 formalizes this discussion. It shows there exists an information structure with $K$ messages that achieves optimality, and the said information structure can be constructed using the sequence of concavifications.

**Proposition 1.** The maximum expected news utility across all information structures is $U^*_1(\pi_0)$. There is an information structure $(M, \sigma)$ with $|M| = K$ attaining this maximum, with the property that after each on-path public history $h^{t-1}$ associated with belief $\pi_{t-1}$, the sender’s strategy $\sigma_t(\cdot \mid h^{t-1}, \theta)$ induces posterior $q^k$ at the end of period $t$ with probability $w^k$, for some $q^1, \ldots, q^K \in \Delta(\Theta)$, $w^1, \ldots, w^K \geq 0$, satisfying $\sum_{k=1}^K w^k = 1$, $\sum_{k=1}^K w^k q^k = \pi_{t-1}$, and $U^*_t(\pi_{t-1}) = \sum_{k=1}^K w^k U_t(q^k \mid \pi_{t-1})$.

A perhaps surprising implication is that the receiver only needs a binary message space if there are two states of the world, regardless of the shape or curvature of the news-utility function $N$. Figure 2 illustrates the concavification procedure in an environment with two equally likely states, $T = 5$, and the mean-based news-utility function $\mu(x) = \sqrt{x}$ for $x \geq 0$, $\mu(x) = -1.5\sqrt{-x}$ for $x < 0$. The sender optimally discloses a conclusive bad-news signal in a random period when $\theta = B$, so each period of silence amounts to a small piece of good news. (In Appendix B.2, we consider Kőszegi and Rabin (2009)’s percentile-based news utility model in a similar environment with Gaussian distributions of residual consumption uncertainty in the two states. We find a very similar optimal information structure under the same square-root gain-loss function.)

The information-design problem imposes additional constraints relative to a habit-formation model. To see this, consider a “relaxed” version of the sender’s problem in the binary-states case where she simply chooses some $x_t \in [0,1]$ each period for $1 \leq t \leq T - 1$, depending on the realization of $\theta$. The receiver gets $\mu_t(x_t - x_{t-1})$ in period $1 \leq t \leq T$, with the initial condition $x_0 = \pi_0$ and the terminal condition $x_T = 1$ if $\theta = G$, $x_T = 0$ if $\theta = B$. One interpretation of the relaxed problem is that the sender chooses the receiver’s sequence of beliefs only subject to the constraint that the initial belief in period 0 is $\pi_0$ and the final
Figure 2: The concavifications giving the optimal information structure with horizon $T = 5$, mean-based news-utility function $\mu(x) = \begin{cases} \sqrt{x} & \text{for } x \geq 0 \\ -1.5\sqrt{-x} & \text{for } x < 0 \end{cases}$, prior $\pi_0 = 0.5$. The dashed vertical line in the $t$-th graph marks the receiver’s belief in $\theta = G$ conditional on not having heard any bad news by the start of period $t$. The $y$-axis shows the sum of news utility this period and the value function of entering next period with a certain belief. In the good state of the world, the receiver’s belief in $\theta = G$ grows at increasing rates across the periods, $0.5 \rightarrow 0.556 \rightarrow 0.626 \rightarrow 0.715 \rightarrow 0.834 \rightarrow 1$. In the bad state of the world, the receiver’s belief follows the same path as in the good state up until the random period when conclusive bad news arrives.
belief in period $T$ puts probability 1 on the true state. The belief paths do not have to be Bayesian. Another interpretation is that $x_t$ is not a belief, but a consumption level for period $t$. The receiver’s welfare in period $t$ only depends on a gain-loss utility based on how current period’s consumption differs from that of period $t-1$. Provided $\mu$ has diminishing sensitivity and exhibits enough loss aversion, the sender maximizes the receiver’s utility by choosing $x_t = \pi_0 + \frac{t}{T}(1 - \pi_0)$ in period $t$ when $\theta = G$, and by choosing $x_t = 0$ in every period $t \geq 1$ when $\theta = B$. The belief paths in Figure 2 differ from these “relaxed” solutions in two ways. First, the receiver gets different amounts of good news (in terms of $\pi_t - \pi_{t-1}$) in different periods when $\theta = G$. Second, the sender sometimes provides false hope in the bad state. These differences come from the Bayesian constraints on beliefs.

### 3.2 Sub-Optimality of One-Shot Resolution

We begin with a sufficient condition on the news-utility function for one-shot resolution to be strictly suboptimal for any $T$ and $\Theta$. Let $\theta_H, \theta_L \in \Theta$ be the states with the highest and lowest consumption utilities. Let $1_H, 1_L \in \Delta(\Theta)$ represent degenerate beliefs in states $\theta_H$ and $\theta_L$ and let $v_0 := \mathbb{E}_{\theta \sim \pi_0}(v(c))$ be the ex-ante expected future consumption utility. The symbol $\oplus$ denotes the mixture between two beliefs in $\Delta(\Theta)$.

**Proposition 2.** For any $T$ and $\Theta$, one-shot resolution is strictly suboptimal if

$$\lim_{\epsilon \to 0^+} \frac{N(1_H | (1-\epsilon)1_H \oplus \epsilon 1_L) - N(1_H | \pi_0) - N(1_L | \pi_0)}{\epsilon} > \lim_{\epsilon \to 0^+} \frac{N(1_H | \pi_0) - N((1-\epsilon)1_H \oplus \epsilon 1_L | \pi_0)}{\epsilon} - N(1_L | 1_H).$$

For the mean-based news-utility model, this condition is equivalent to

$$\mu'(0^+) + \mu(1-v_0) - \mu(-v_0) > \mu'(1-v_0) - \mu(-1).$$

In fact, the proof of Proposition 2 shows that whenever its condition is satisfied, some information structure featuring gradual good news and one-shot bad news (to be defined precisely in the next subsection) is strictly better than one-shot resolution.

We can interpret Proposition 2’s sufficient condition as “strong enough diminishing sensitivity.” Evidently, $\mu(1-v_0) - \mu(-v_0) > 0$, so the condition is satisfied whenever $\mu'(0^+) \geq \mu'(1-v_0) - \mu(-1)$. If $\mu'(0^+)$ is sufficiently large or $\mu''$ is sufficiently negative
to the right of 0 and sufficiently positive to the left of 0, then both of the positive RHS terms \( \mu'(1 - v_0) \) and \(-\mu(-1)\) will be small relative to \( \mu'(0^+) \), thus the inequality will hold.

The quadratic news utility provides a clear illustration of this interpretation, as the condition of Proposition 2 holds whenever there is enough curvature relative to the extent of loss aversion.

**Corollary 1.** If the receiver has quadratic news utility with \( \alpha_n - \alpha_p \leq \beta_n + \beta_p \), then one-shot resolution is strictly suboptimal for any \( T \).

The difference \( \alpha_n - \alpha_p \geq 0 \) is the size of the “kink” at 0, that is \( \mu'(0^-) - \mu'(0^+) \). On the other side, \( \beta_p \) and \( \beta_n \) control the amounts of curvature in the positive and negative regions, respectively.

The sufficient condition in Proposition 2 is also satisfied by the most commonly used model of diminishing sensitivity, the power function (see, for example, Tversky and Kahneman (1992)). One could think of the power function specification as having “infinite” diminishing sensitivity near 0, as \( \mu''(0^+) = -\infty \) and \( \mu''(0^-) = \infty \).

**Corollary 2.** Suppose \( \mu(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda \cdot |x|^\beta & \text{if } x < 0 \end{cases} \) for some \( 0 < \alpha, \beta < 1 \) and \( \lambda \geq 1 \). Then one-shot resolution is strictly suboptimal for any \( T \).

While Proposition 2 holds generally, we can find sharper results on the sub-optimality of one-shot resolution for specific news-utility models and environments. Kőszegi and Rabin (2009)’s percentile-based news-utility model stipulates

\[
N(\pi_t | \pi_{t-1}) = \int_0^1 \mu \left( v(F_{\pi_t}(p)) - v(F_{\pi_{t-1}}(p)) \right) dp,
\]

where \( F_{\pi_t}(p) \) and \( F_{\pi_{t-1}}(p) \) are the \( p \)-th percentile consumption levels according to beliefs \( \pi_t \) and \( \pi_{t-1} \), respectively. Whenever \( \mu \) exhibits diminishing sensitivity to gains and there are at least 3 states, one-shot resolution is suboptimal. This result does not require any assumption about loss aversion.

**Proposition 3.** In Kőszegi and Rabin (2009)’s percentile-based news-utility model, provided the gain-loss utility function satisfies \( \mu''(x) < 0 \) for all \( x > 0 \), one-shot resolution is strictly suboptimal for any \( T \) and any \( K \geq 3 \).
Similar to the idea behind Proposition 2, the proof of Proposition 3 constructs an information structure to gradually deliver the good news that the state is the best one possible. By contrast, if \( \mu \) is two-part linear with \( \mu(x) = bx \) for \( x \geq 0 \), \( \mu(x) = \lambda bx \) for some \( b > 0 \), \( \lambda > 1 \) (so that \( \mu''(x) = 0 \) for \( x > 0 \)), then one-shot resolution is the uniquely optimal information structure (Kőszegi and Rabin, 2009).

Proposition 3 requires at least three distinct consumption levels, \( K \geq 3 \). In a binary-states world, the percentile-based news-utility function \( N \) only depends on the value of \( \mu \) at two non-zero points. Thus every increasing \( \mu \) is behaviorally indistinguishable from a two-part linear one, meaning the percentile-based model cannot capture diminishing sensitivity in a setting with \( K = 2 \).

As an analog to Corollary 2, we study a setting with percentile-based news utility and residual consumption uncertainty in Appendix B, finding that one-shot resolution is strictly suboptimal with any number of states for a power-function \( \mu \) (Corollary A.1).

### 3.3 Gradual Good News and Gradual Bad News

For the remainder of the paper, we focus on mean-based news-utility functions to study additional implications of diminishing sensitivity. Two classes of information structures will play important roles in the sequel. To define them, we write \( v_t := \mathbb{E}_{\theta \sim \pi_t}[v(c_\theta)] \) for the expected future consumption utility based on the receiver’s (random) belief at the end of period \( t \). Partition states into two subsets, \( \Theta = \Theta_B \cup \Theta_G \), where \( v(c_\theta) < v_0 \) for \( \theta \in \Theta_B \) and \( v(c_\theta) \geq v_0 \) for \( \theta \in \Theta_G \). Interpret \( \Theta_B \) as the “bad” states and \( \Theta_G \) as the “good” ones.

**Definition 2.** An information structure \((M, \sigma)\) features **gradual good news, one-shot bad news** if

- \( \mathbb{P}_{(M, \sigma)}[v_t \geq v_{t-1} \text{ for all } 1 \leq t \leq T \mid \theta \in \Theta_G] = 1 \) and
- \( \mathbb{P}_{(M, \sigma)}[v_t < v_{t-1} \text{ for no more than one } 1 \leq t \leq T \mid \theta \in \Theta_B] = 1. \)

An information structure \((M, \sigma)\) features **gradual bad news, one-shot good news** if

- \( \mathbb{P}_{(M, \sigma)}[v_t \leq v_{t-1} \text{ for all } 1 \leq t \leq T \mid \theta \in \Theta_B] = 1 \) and
- \( \mathbb{P}_{(M, \sigma)}[v_t > v_{t-1} \text{ for no more than one } 1 \leq t \leq T \mid \theta \in \Theta_G] = 1. \)
In the first class of information structures (“gradual good news, one-shot bad news”), the sender relays good news over time and gradually increases the receiver’s expectation of future consumption. When the state is bad, the sender concentrates all the bad news in one period. The “one-shot bad news” terminology comes from noting that when $\theta \in \Theta_B$, the single period $t$ where $v_t < v_{t-1}$ must satisfy $v_t = v(\theta)$ and $v_{t'} = v_t$ for all $t' > t$. The receiver gets negative information about his future consumption level for the first time in period $t$, and his expectation stays constant thereafter. On the other hand, we use the phrase “gradual bad news, one-shot good news” to refer to the “opposite” kind of information structure.

One-shot resolution falls into both of these classes. To rule out this triviality, we say that an information structure features strictly gradual good news if

$$P_{(M,\sigma)}[v_t > v_{t-1} \text{ and } v_{t'} > v_{t'-1} \text{ for two distinct } 1 \leq t, t' \leq T \mid \theta \in \Theta_G] > 0.$$ 

That is, there is positive probability that the receiver’s expectation strictly increases at least twice in periods 1 through $T$. Similarly define strictly gradual bad news.

We now prove that whenever $\mu$ satisfies diminishing sensitivity and (weak) loss aversion, information structures featuring strictly gradual bad news, one-shot good news are strictly worse than one-shot resolution. By contrast, under some additional restrictions, the optimal information structure falls into the strictly gradual good news, one-shot bad news class.

**Proposition 4.** Suppose $\mu$ satisfies diminishing sensitivity and loss aversion. Any information structure featuring strictly gradual bad news, one-shot good news is strictly worse than one-shot resolution in expectation, and almost surely weakly worse ex-post.

This result holds for arbitrary state space $\Theta$, horizon $T$, and prior $\pi_0$.

For the rest of the paper, we specialize to the case of $K = 2$. The next result presents a necessary and sufficient condition for inconclusive bad news to be suboptimal when $T = 2$. We then verify the condition for quadratic news utility.

**Proposition 5.** For $T = 2$, information structures with $P_{(M,\sigma)}[\pi_1 < \pi_0 \text{ and } \pi_1 \neq 0] > 0$ are strictly suboptimal if and only if there exists some $q \geq \pi_0$ so that the chord connecting $(0, U_1(0 \mid \pi_0))$ and $(q, U_1(q \mid \pi_0))$ lies strictly above $U_1(p \mid \pi_0)$ for all $p \in (0, \pi_0)$.

**Corollary 3.** Quadratic news utility satisfies the condition of Proposition 5.
In particular, combining Corollaries 1 and 3, we infer that any optimal information structure for a receiver with quadratic news utility with \( \alpha_n - \alpha_p \leq \beta_n + \beta_p \) with \( T = 2 \) must feature strictly gradual good news, one-shot bad news. Furthermore, since there exists an optimal information structure with binary messages by Proposition 1, in this environment there is an optimal information structure where the sender induces either belief 0 or belief \( p_H > \pi_0 \) in the only period of communication. The next subsection characterizes \( p_H \) as a function of the model parameters.

In summary, we have established a ranking between three kinds of information structures. For any time horizon and any state space, provided the condition in Proposition 2 holds and \( \mu \) satisfies diminishing sensitivity and weak loss aversion, some information structure featuring gradual good news, one-shot bad news gives more news utility than one-shot resolution, which in turn gives more news utility than any information structure featuring strictly gradual bad news, one-shot good news. Further, under the additional restrictions in Proposition 5, a gradual good news, one-shot bad news information structure is optimal among all information structures.

3.4 Illustrative Example: Quadratic News Utility

We illustrate Proposition 1’s concavification procedure by finding in closed-form the optimal information structure when the receiver has a quadratic news-utility function.

Suppose the parameters of \( \mu \) satisfy \( \alpha_n - \alpha_p \leq \beta_n + \beta_p \) in a \( T = 2 \) environment. From the arguments in Section 3.3, the optimal information structure induces either \( \pi_1 = 0 \) or \( \pi_1 = p_H \) for some \( p_H > \pi_0 \). Proposition 1 implies \( (\text{cav}U_1(\cdot | \pi_0))(x) > U_1(x | \pi_0) \) for all \( x \in (0, p_H) \). The geometry of concavification shows the derivative of the value function at \( p_H \), \( \frac{d}{dx} U_1(x | \pi_0)(p_H) \), equals the slope of the chord from 0 to \( p_H \) on the function \( U_1(\cdot | \pi_0) \). We use this equality to derive \( p_H \) as the solution to a cubic polynomial.

**Proposition 6.** For \( T = 2 \) and quadratic news utility satisfying \( \alpha_n - \alpha_p \leq \beta_n + \beta_p \), the optimal partial good news \( p_H > \pi_0 \) satisfies

\[
\pi_0 (\alpha_n - \alpha_p) - (\beta_p + \beta_n) \pi_0^2 = p_H^2 (\alpha_n - \alpha_p + \beta_n + \beta_p) - p_H^3 (2\beta_p + 2\beta_n).
\]

We have \( \frac{dp_H}{d\pi_0} > 0 \) for \( \pi_0 < \frac{1}{2} \frac{\alpha_n - \alpha_p}{\beta_n + \beta_p} \) and \( \frac{dp_H}{d\pi_0} < 0 \) for \( \pi_0 > \frac{1}{2} \frac{\alpha_n - \alpha_p}{\beta_n + \beta_p} \).
Figure 3: Optimal partial good news with quadratic news utility and $T = 2$, fixing parameters $\alpha_p = 2$, $\alpha_n = 2.1$, $\beta_p = 1$, $\beta_n = 0.2$ and considering different prior beliefs. The dashed blue line is at $\pi_0 = \frac{1}{2} \frac{\alpha_n - \alpha_p}{\beta_n + \beta_p} \approx 0.042$. The optimal partial good news is decreasing in the prior before this threshold, and increasing afterwards.

In other words, the optimal partial good news is in general non-monotonic in the prior belief. For low prior beliefs, $p_H$ increases with prior. But for high prior beliefs, $p_H$ decreases with prior. Figure 3 illustrates. In the case of $\alpha_n = \alpha_p$, and in particular when $\mu$ is symmetric around 0, $\frac{d\mu}{d\pi_0} > 0$ for any $\pi_0 \in (0, 1)$.

3.5 Endogenous Diversity of Information Preferences

Leaving aside the setting where the sender knows the state upfront and can choose any information structure, consider an environment where a sequence of exogenous signal realizations determine the state. We show that agents with opposite consumption preferences over the two states can exhibit opposite preferences when choosing between observing the signals as they arrive (gradual resolution) or only learning the final state (one-shot resolution).

There are two states of the world, Alternative ($A$) and Baseline ($B$). In each period $t = 1, 2, ..., T$, a binary random variable $X_t$ realizes, where $\mathbb{P}[X_t = 1] = q_t$ with $0 < q_t < 1$. If $X_t = 1$ for all $t$, then the state is $A$. Else, if $X_t = 0$ for at least one $t$, then the state is $B$. The agent’s consumption utility in period $T$ depends on the state, and is normalized without loss to be either 0 or 1.
At time 0, the agent chooses between observing the realizations of the random variables \( (X_t)_{t=1}^T \) in real time, or only learning the state of the world at the end of period \( T \). As an example, imagine a televised debate between two political candidates \( A \) and \( B \) where \( A \) loses as soon as she makes a “gaffe” during the debate.\(^3\) If \( A \) does not make any gaffes, then \( A \) wins. An individual who strongly prefers one of the candidates to win must choose between watching the debate live in the evening or only reading the outcome of the debate the following morning.

The agent forms Bayesian belief \( \pi_t \in [0,1] \) about the probability of state \( A \) at the end of each period \( t \), starting with the correct Bayesian prior \( \pi_0 \). For notational convenience, we also write \( \rho_t = 1 - \pi_t \) as the belief in state \( B \) at the end of \( t \), with the prior \( \rho_0 = 1 - \pi_0 \). If the agent prefers state \( A \), he gets news utility \( \mu(\pi_t - \pi_{t-1}) \) at the end of period \( t \). If the agent prefers state \( B \), then he gets news utility \( \mu(\rho_t - \rho_{t-1}) \). The function \( \mu \) exhibits diminishing sensitivity, that is \( \mu''(x) < 0 \) and \( \mu''(-x) > 0 \) for \( x > 0 \). Also, to quantify the amount of loss aversion, we consider the parametric class of \( \lambda \)-scaled news-utility functions. We fix some \( \tilde{\mu}_{pos} : [0,1] \to \mathbb{R}_+ \), strictly increasing and strictly concave with \( \tilde{\mu}_{pos}(0) = 0 \), and consider the family of \( \mu \)'s given by \( \mu(x) = \tilde{\mu}_{pos}(x), \mu(-x) = -\lambda \tilde{\mu}_{pos}(x) \) for \( x > 0 \) as we vary \( \lambda \geq 1 \).

Under diminishing sensitivity, someone rooting for state \( A \) wants to watch the events unfold in real time to celebrate the small victories, while someone hoping for state \( B \) prefers to only learn the final state to avoid piecemeal bad news. The next proposition formalizes this intuition.

**Proposition 7.** Consider the class of \( \lambda \)-scaled news-utility functions. For any \( \lambda \geq 1 \), an agent who prefers state \( B \) will choose one-shot resolution of uncertainty over gradual resolution of uncertainty. There exists some \( \bar{\lambda} > 1 \) so that for any \( 1 \leq \lambda \leq \bar{\lambda} \), an agent who prefers state \( A \) will choose gradual resolution of uncertainty over one-shot resolution of uncertainty.

This result suggests a possible mechanism for media competition: if the realization of some state \( A \) depends on a series of smaller events, then some news sources may cover these small events in detail as they happen, while other sources may choose to only report the final outcome. Viewers sort between these two kinds of news sources based on how they rank

\(^3\)Augenblick and Rabin (2018) use a similar example of political gaffes to illustrate Bayesian belief movements.
states $A$ and $B$ in terms of consumption. Opposite consumption preferences induce opposite informational preferences.

By contrast, other behavioral models do not predict a diversity of informational preferences in this environment.

**Proposition 8.** The following models do not predict different informational preferences for agents with the opposite consumption rankings for the two states.

1. Two-part linear news-utility function $\mu$.

2. Anticipatory utility where the agent gets either $u(\pi_t)$ or $u(\rho_t)$ in period $t$ depending on his preference over states $A$ and $B$, with $u$ an increasing, weakly concave function.


Another application of Proposition 7 concerns the design of game shows. Consider a game show featuring a single contestant who will win either $100,000 or nothing depending on her performance across five rounds. The audience, empathizing with the contestant, derives news utility $\mu(\pi_t - \pi_{t-1})$ at the end of round $t$, where $\pi_t$ is the contestant’s probability of winning the prize based on the first $t$ rounds. One possible format (“sudden death”) features five easy rounds each with $w = 0.5^{1/5} \approx 87\%$ winning probability, where the contestant wins $100,000 if she wins all five rounds. Another possible format (“repêchage”) involves five hard rounds each with $1 - w$ winning probability, but the contestant wins $100,000 as soon as she wins any round. Both formats lead to the same distribution over final outcomes and generate the same amount of suspense and surprise utilities à la Ely, Frankel, and Kamenica (2015). Proposition 7 shows the first format induces more news utility than one-shot resolution for audience members who are not too loss averse, while the second format is worse than one-shot resolution for all audience members. Consistent with our model, the vast majority of game shows resemble the first format more than the second format.

4 The Credibility Problem of Gradual Good News

Section 3 studied the optimal disclosure of news when the sender has commitment power. We provided conditions for the optimal information structure to feature gradual good news,
one-shot bad news. Information structures of this kind encounter a credibility problem when the commitment assumption is dropped. If the sender wishes to gradually reveal the good state to a Bayesian receiver over multiple periods, then she must also sometimes provide false hope in the bad state due to the cross-state constraints on beliefs. But without commitment, the benevolent sender may strictly prefer giving false hope over telling the truth in the bad state. This deviation improves the total news utility of a receiver with diminishing sensitivity, if the positive utility from today’s good news outweighs the additional future disappointment from higher expectations. In fact, when news utility is symmetric and exhibits diminishing sensitivity, the above credibility problem is so severe that every equilibrium is payoff-equivalent to the babbling equilibrium. The same result also applies to asymmetric news-utility functions with \( \mu(-x) = -\lambda \mu(x) \) for all \( x > 0 \), provided loss aversion \( \lambda > 1 \) is weak enough relative to \( \mu \)'s diminishing sensitivity in a way we formalize.

Sufficiently strong loss aversion can restore the equilibrium credibility of good-news messages. We show that the highest equilibrium payoff when the sender lacks commitment may be non-monotonic in the extent of loss aversion, in contrast to the conclusion that more loss-averse receivers are always strictly worse off when the sender has commitment power. We also completely characterize the class of equilibria that feature (a deterministic sequence of) gradual good news in the good state and study the equilibrium rate of learning. With the quadratic or the square-root news-utility function, equilibria within this class always release progressively larger pieces of good news over time, so the receiver’s belief in the good state grows at an increasing rate.

4.1 Equilibrium Analysis When the Sender Lacks Commitment

We continue to maintain that state space \( \Theta = \{G, B\} \) is binary. To study the case where the sender lacks commitment, we analyze the perfect-Bayesian equilibria of the cheap talk game between the two parties. Formally, the equilibrium concept is as follows.

**Definition 3.** Let a finite set of messages \( M \) be fixed. A **perfect-Bayesian equilibrium** consists of sender’s strategy \( \sigma^* = (\sigma^*_t)_{t=1}^{T-1} \) together with receiver’s beliefs \( p^* : \cup_{t=0}^{T-1} H^t \to [0, 1] \), where:

- For every \( 1 \leq t \leq T - 1 \), \( h^{t-1} \in H^{t-1} \) and \( \theta \in \{G, B\} \), \( \sigma^* \) maximizes the receiver’s total expected news utility in periods \( t, ..., T - 1, T \) conditional on having reached the
public history \( h^{t-1} \) in state \( \theta \) at the start of period \( t \).

- \( p^\ast \) is derived by applying the Bayes’ rule to \( \sigma^\ast \) whenever possible.

We make two belief-refinement restrictions:

- If \( t \leq T - 1 \), \( h^t \) is a continuation history of \( h^t \), and \( p^\ast(h^t) \in \{0, 1\} \), then \( p^\ast(h^t) = p^\ast(h^t) \).
- The receiver’s belief in period \( T \) when state is \( \theta \) satisfies \( \pi_T = 1_\theta \), regardless of the preceding history \( h^{T-1} \in H^{T-1} \).

We will abbreviate a perfect-Bayesian equilibrium satisfying our belief refinements as an “equilibrium.” Our definition requires that once the receiver updates his belief to 0 or 1, it stays constant through the end of period \( T - 1 \). In period \( T \), the receiver updates his belief to reflect full confidence in the true state of the world, regardless of his (possibly dogmatic) belief at the end of period \( T - 1 \). The receiver derives news utility in periods \( 1 \leq t \leq T \) based on changes in his belief, as in the model with commitment.

Let \( V_{\mu,M,T}(\pi_0) \subseteq \mathbb{R} \) denote the set of equilibrium payoffs with news-utility function \( \mu \), message space \( M \), time horizon \( T \), and prior \( \pi_0 \). Clearly, \( V_{\mu,M,T}(\pi_0) \) is non-empty. There is always the babbling equilibrium, where the sender mixes over all messages uniformly in both states and the receiver’s belief never updates from the prior belief until period \( T \). Denote the babbling equilibrium payoff by

\[
V^{Bab}_{\mu}(\pi_0) := \pi_0 \mu(1 - \pi_0) + (1 - \pi_0) \mu(-\pi_0)
\]

and note it is independent of \( M \) or \( T \).

We state two preliminary properties of the equilibrium payoffs set \( V_{\mu,M,T}(\pi_0) \).

**Lemma 1.** We have:

1. For any finite \( M \), \( V_{\mu,M,T}(\pi_0) \subseteq V_{\mu,\{g,b\},T}(\pi_0) \)
2. If \( T \leq T' \), then \( V_{\mu,M,T}(\pi_0) \subseteq V_{\mu,M,T'}(\pi_0) \).

The first statement says any equilibrium payoff achievable with an arbitrary finite message space is also achievable with a binary message space. The second statement says the set of equilibrium payoffs weakly expands with the time horizon.
4.2 The Credibility Problem and Babbling

To understand the source of the credibility problem, let $N_B(x; \pi) := \mu(x - \pi) + \mu(-x)$ denote the total amount of news utility across two periods when the receiver updates his belief from $\pi$ to $x > \pi$ today and updates it from $p$ to 0 tomorrow. Suppose there exists a period $T - 2$ public history $h^{T-2} \in H^{T-2}$ with $p^*(h^{T-2}) = \pi$ and some $x > \pi$ satisfying $N_B(x; \pi) > N_B(0; \pi)$. Then, the sender strictly prefers to induce belief $x$ rather than belief 0 after arriving at the history $h^{T-2}$ in the bad state. A good-news message $m_x$ inducing belief $x$ and a bad-news message $m_0$ inducing belief 0 cannot both be on-path following $h^{T-2}$, else the sender would strictly prefer to send $m_x$ with probability 1 in the bad state.

Yet, the inequality $N_B(0; \pi) < N_B(x; \pi)$ automatically holds for any $x > \pi$, provided $\mu$ is strictly concave in the positive region and symmetric around 0.

**Lemma 2.** If $\mu$ is symmetric around 0 and $\mu''(x) < 0$ for all $x > 0$, then for any $0 < \pi < x < 1$ it holds $N_B(0; \pi) < N_B(x; \pi)$.

The intuition is that when the state is bad, the sender knows the receiver will inevitably get conclusive bad news in period $T$. Giving false hope in period $T - 1$ (i.e., inducing belief $x > \pi$ instead of 0) provides positive news utility at the cost of greater disappointment in the final period. Diminishing sensitivity limits the incremental cost of this additional disappointment.

The credibility problem implies that the babbling payoff is the unique equilibrium payoff.

**Proposition 9.** Suppose $\mu$ is symmetric around 0 and $\mu''(x) < 0$ for all $x > 0$. For any $M, T, \pi_0$, $\mathcal{V}_{\mu, M, T}(\pi_0) = \{V_{\mu}^{Bab}(\pi_0)\}$.

We now explore what happens when $\mu$ is asymmetric around 0 due to loss aversion. Say $\mu$ exhibits greater sensitivity to losses if $\mu'(x) \leq \mu'(-x)$ for all $x > 0$. We first establish a robustness check on Proposition 9 within this class of news-utility functions: when loss aversion is sufficiently weak relative to diminishing sensitivity in a $T = 2$ model, the babbling equilibrium remains unique up to payoffs.

**Proposition 10.** Suppose $\mu$ exhibits greater sensitivity to losses. If $\min_{z \in [0, 1 - \pi_0]} \frac{\mu'(z)}{\mu'(-\pi_0 + z)} > 1$, then $\mathcal{V}_{\mu, M, 2}(\pi_0) = \{V_{\mu}^{Bab}(\pi_0)\}$ for any $M$.

When $\mu$ is symmetric and does not exhibit strict loss aversion, diminishing sensitivity implies $\mu'(-\pi_0 + z) = \mu'(\pi_0 + z) < \mu'(z)$ for every $z \in [0, 1 - \pi_0]$, so the inequality
condition in Proposition 10 is always satisfied. This condition continues to hold if \( \mu \) is slightly asymmetric due to a “small enough” amount of loss aversion relative to the size of the sensitivity gap \( \mu'(z) - \mu'(\pi_0 + z) \). This interpretation is clearest for the \( \lambda \)-scaled news-utility functions, as formalized in the following corollary.

**Corollary 4.** Suppose for some \( \tilde{\mu}_{\text{pos}} : [0, 1] \to \mathbb{R}_+ \) and \( \lambda \geq 1 \), the news-utility function \( \mu \) satisfies \( \mu(x) = \tilde{\mu}_{\text{pos}}(x) \), \( \mu(-x) = -\lambda \tilde{\mu}_{\text{pos}}(x) \) for all \( x \geq 0 \). Provided \( \lambda < \min_{z \in [0, 1-\pi_0]} \frac{\tilde{\mu}'_{\text{pos}}(z)}{\tilde{\mu}'_{\text{pos}}(\pi_0 + z)} \), \( V_{\mu, M, T}(\pi_0) = \{ V^{\text{Bab}}_{\mu}(\pi_0) \} \) for any \( M \).

When \( \mu \) is strictly concave in the positive region, Corollary 4 gives a non-degenerate interval of loss-aversion parameters for which the conclusion of Proposition 9 extends in a \( T = 2 \) setting. If \( \tilde{\mu}_{\text{pos}} \) contains more curvature, then \( \tilde{\mu}'_{\text{pos}}(z)/\tilde{\mu}'_{\text{pos}}(\pi_0 + z) \) becomes larger and the interval of permissible \( \lambda \)'s expands.

What happens when loss aversion is high? The next proposition says a new equilibrium that payoff-dominates the babbling one exists for large \( \lambda \), provided the marginal utility of an infinitesimally small piece of good news is infinite — as in the power-function specification.

**Proposition 11.** Fix \( \tilde{\mu}_{\text{pos}} : [0, 1] \to \mathbb{R}_+ \) strictly increasing and concave, continuously differentiable at \( x > 0 \), \( \tilde{\mu}_{\text{pos}}(0) = 0 \), and \( \lim_{x \to 0} \tilde{\mu}'_{\text{pos}}(x) = \infty \). Consider the family \( \lambda \)-indexed news-utility functions \( \mu(x) = \tilde{\mu}_{\text{pos}}(x) \), \( \mu(-x) = -\lambda \tilde{\mu}_{\text{pos}}(x) \) for \( x \geq 0 \). For each \( \pi_0 \in (0, 1) \), there exists \( \tilde{\lambda} \geq 1 \) so that whenever \( \lambda \geq \tilde{\lambda} \) and for any \( T \geq 2 \), \( |M| \geq 2 \), there exists \( V \in V_{\mu, M, T}(\pi_0) \) with \( V > V^{\text{Bab}}_{\mu}(\pi_0) \).

To help illustrate these results, suppose \( \mu(x) = \sqrt{x} \) for \( x \geq 0 \), \( \mu(x) = -\lambda \sqrt{-x} \) for \( x < 0 \), \( T = 2 \), and \( \pi_0 = \frac{1}{2} \). Corollary 4 implies whenever \( \lambda < \sqrt{2} \), the babbling equilibrium is unique up to payoffs. On the other hand, Proposition 11 says when \( \lambda \) is sufficiently high, there is another equilibrium with strictly higher payoffs. In fact, a non-babbling equilibrium first appears when \( \lambda = 2.414 \).

Figure 4 plots the highest equilibrium payoff for different values of \( \lambda \). Receivers with higher \( \lambda \) may enjoy higher equilibrium payoffs. The reason for this non-monotonicity is that for low values of \( \lambda \), the babbling equilibrium is unique and increasing \( \lambda \) decreases expected news utility linearly. When the new, non-babbling equilibrium emerges for large enough \( \lambda \), the sender’s behavior in the new equilibrium depends on \( \lambda \). Higher loss aversion carries two countervailing effects: first, a non-strategic effect of hurting welfare when \( \theta = B \), as the receiver must eventually hear the bad news; second, an equilibrium effect of changing the
Figure 4: The babbling equilibrium is essentially unique for low values of $\lambda$, but there exists an equilibrium with gradual good news for $\lambda \geq 2.414$. Due to the role of loss aversion in sustaining credible partial news, a receiver with higher loss aversion may experience higher or lower expected news utility in equilibrium than a receiver with lower loss aversion.

relative amounts of good news in different periods conditional on $\theta = G$. Receivers with an intermediate amount of loss aversion enjoy higher expected news utility than receivers with low loss aversion, as the equilibrium effect leads to better “consumption smoothing” of good news across time. But, the non-strategic effect eventually dominates and receivers with high loss aversion experience worse payoffs than receivers with low loss aversion.

4.3 Deterministic Gradual Good News Equilibria

An equilibrium $(M, \sigma^*, p^*)$ features deterministic\(^5\) gradual good news (GGN equilibrium) if there exist a sequence of constants $p_0 \leq p_1 \leq \ldots \leq p_{T-1} \leq p_T$ with $p_0 = \pi_0$, $p_T = 1$, and the receiver always has belief $p_t$ in period $t$ when the state is good. By Bayesian beliefs, in the bad state of any GGN equilibrium the sender must induce a belief of either 0 or $p_t$ in period $t$, as any message not inducing belief $p_t$ is a conclusive signal of the bad state.

The class of GGN equilibria is non-empty, for it contains the babbling equilibrium where $\pi_0 = p_0 = p_1 = \ldots = p_{T-1} < p_T$. The number of intermediate beliefs in a GGN equilibrium is the number of distinct beliefs in the open interval $(\pi_0, 1)$ along the sequence $p_0, p_1, \ldots, p_{T-1}$.

\(^5\)This class of equilibria is slightly more restrictive than the gradual good news, one-shot bad news information structures from Definition 2, because the sender may not randomize between several increasing paths of beliefs in the good state.
The babbling equilibrium has zero intermediate beliefs.

The next proposition characterizes the set of all GGN equilibria with at least one intermediate belief.

**Proposition 12.** Let $P^*(\pi) \subseteq (\pi, 1]$ be those beliefs $x$ satisfying $N_B(x; \pi) = N_B(0; \pi)$. Suppose $\mu$ exhibits diminishing sensitivity and loss aversion. For $1 \leq J \leq T - 1$, there exists a gradual good news equilibrium with the $J$ intermediate beliefs $q^{(1)} < \ldots < q^{(J)}$ if and only if $q^{(j)} \in P^*(q^{(j-1)})$ for every $j = 1, \ldots, J$, where $q^{(0)} := \pi_0$.

To interpret, $P^*(\pi)$ contains the set of beliefs $x > \pi$ such that the sender is indifferent between inducing the two belief paths $\pi \rightarrow x \rightarrow 0$ and $\pi \rightarrow 0$. Recall that when $\mu$ is symmetric, Lemma 2 implies this indifference condition is never satisfied, which is the source of the credibility problem for good-news messages. The same indifference condition pins down the relationship between successive intermediate beliefs in GGN equilibria.

We illustrate this result with the quadratic news utility.

**Corollary 5.** 1) With quadratic news utility, $P^*(\pi) = \left\{ \pi \cdot \frac{\beta_p + \beta_n}{\beta_p - \beta_n} - \frac{\alpha_n - \alpha_p}{\beta_p - \beta_n} \right\} \cap (\pi, 1)$.

2a) If $\beta_n > \beta_p$, there cannot exist any gradual good news equilibrium with more than one intermediate belief.

2b) If $\beta_n < \beta_p$, there can exist gradual good news equilibria with more than one intermediate belief. For a given set of parameters of the quadratic news-utility function and prior $\pi_0$, there exists a uniform bound on the number of intermediate beliefs that can be sustained in equilibrium across all $T$.

3) In any GGN equilibrium with quadratic news utility, intermediate beliefs in the good state grow at an increasing rate.

Combined with Proposition 12, part 1) of this corollary says that in every GGN equilibrium, the successive intermediate beliefs are related by the linear map $x \mapsto x \cdot \frac{\beta_p + \beta_n}{\beta_p - \beta_n} - \frac{\alpha_n - \alpha_p}{\beta_p - \beta_n}$. When $\beta_n > \beta_p$, this map has a negative slope, so there cannot exist any GGN equilibrium with more than one intermediate belief. When $\beta_p > \beta_n$, this map has a slope strictly larger than 1. As a result, after eliminating periods where no informative signal is released, every GGN equilibrium releases progressively larger pieces of good news in the good state, $q^{(j+1)} - q^{(j)} > q^{(j)} - q^{(j-1)}$. Since equilibrium beliefs in the good state grow at an increasing rate, there exists some uniform bound $\bar{J}$ on the number of intermediate beliefs depending only on the prior belief $\pi_0$ and parameters of the news-utility function.
Beliefs in GGN equilibrium, $\alpha_p = 2$, $\alpha_n = 2.1$, $\beta_p = 1$, $\beta_n = 0.2$

Figure 5: The longest possible sequence of GGN intermediate beliefs starting with prior $\pi_0 = \frac{1}{3}$. For quadratic news utility, equilibrium GGN beliefs always increase at an increasing rate in the good state.

As an illustration, consider the quadratic news utility with $\alpha_p = 2$, $\alpha_n = 2.1$, $\beta_p = 1$, and $\beta_n = 0.2$. Starting at the prior belief of $\pi_0 = \frac{1}{3}$, Figure 5 shows the longest possible sequence of intermediate beliefs in any GGN equilibrium for arbitrarily large $T$. Since the $P^*$ sets are either empty sets or singleton sets for the quadratic news utility, Figure 5 also contains all the possible beliefs in any state of any GGN equilibrium with these parameters.

The result that GGN equilibria release increasingly larger pieces of good news generalizes to other news-utility functions with diminishing sensitivity. The basic intuition is that if the sender is indifferent between providing $d$ amount of false hope and truth-telling in the bad state when the receiver has prior belief $\pi_L$ (i.e., $\pi_L + d \in P^*(\pi_L)$), then she strictly prefers providing the same amount of false hope over truth-telling at any higher prior belief $\pi_H > \pi_L$. The false hope generates the same positive news utility in both cases, but an extra $d$ units of disappointment matters less when added a baseline disappointment level of $\pi_H$ rather than $\pi_L$, thanks to diminishing sensitivity.

The next proposition formalizes this idea. It shows that when diminishing sensitivity is combined with a pair of regularity conditions, intermediate beliefs grow at an increasing rate in any GGN equilibrium.

**Proposition 13.** Suppose $\mu$ exhibits diminishing sensitivity, $|P^*(\pi)| \leq 1$ and $\frac{\partial}{\partial x} N_B(x; \pi)|_{x=\pi} > 0$ for all $\pi \in (0, 1)$. Then, in any GGN equilibrium with intermediate beliefs $q^{(1)} < \ldots < q^{(J)}$, we get $q^{(j)} - q^{(j-1)} < q^{(j+1)} - q^{(j)}$ for all $1 \leq j \leq J - 1$.  

28
The first regularity condition requires that the sender is indifferent between the belief paths \( \pi \to x \to 0 \) and \( \pi \to 0 \) for at most one \( x > \pi \). It is a technical assumption that lets us prove our result, but we suspect the conclusion also holds under some relaxed conditions. The second regularity condition says in the bad state, the total news utility associated with an \( \epsilon \) amount of false hope is higher than truth-telling for small \( \epsilon \). These conditions are satisfied by the power-function news utility with \( \alpha = \beta \), for example.

**Corollary 6.** In any GGN equilibrium with power-function news utility with \( \alpha = \beta \) and any \( \lambda \geq 1 \), intermediate beliefs in the good state grow at an increasing rate.

## 5 A Random-Horizon Model

In this section, we study a version of our information design problem without a deterministic horizon. Each period, with probability \( 1 - \delta \in (0, 1] \), the true state of the world is exogenously revealed to the receiver and the game ends. Until then, the informed sender communicates with the receiver each period as in the model from Section 2. We verify that our results from the finite-horizon setting extend analogously into this random-horizon environment.

### 5.1 The Random-Horizon Model

Consider an environment where the consumption event takes place far in the future, but the sender is no longer the receiver’s only source of information in the interim. Instead, a third party perfectly discloses the state to the receiver with some probability each period. For instance, the sender may be the chair of a central bank who has decided on the bank’s monetary policy for next year and wishes to communicate this information over time, while the third party is an employee of the bank who also knows the planned policy. With some probability each period, the employee goes to the press and leaks the future policy decision.

Time is discrete with \( t = 0, 1, 2, \ldots \). The sender commits to an information structure \((M, \sigma)\) at time 0. The information structure consists of a finite message space \( M \) and a sequence of message strategies \( (\sigma_t)_{t=1}^{\infty} \) where each \( \sigma_t(\cdot \mid h_{t-1}, \theta) \in \Delta(M) \) specifies how the sender will mix over messages in period \( t \) as a function of the public history \( h_{t-1} \) so far and the true state \( \theta \).

The sender learns the state at the beginning of period 1 and sends a message according
to $\sigma_t$. At the start of each period $t = 2, 3, 4, \ldots$, there is probability $(1 - \delta) \in (0, 1]$ that the receiver exogenously and perfectly learns the state $\theta$. If so, the game effectively ends because no further communication from the sender can change the receiver’s belief. If not, then the sender sends the next message according to $\sigma_t$. The randomization over exogenous learning is i.i.d. across periods, so the time of state revelation (i.e., the horizon of the game) is a geometric random variable.

5.2 The Value Function with Commitment

Let $V_\delta : [0, 1] \rightarrow \mathbb{R}$ be the value function of the problem with continuation probability $\delta$ — that is, $V_\delta(p)$ is the highest possible total expected news utility up to the period of state revelation, when the receiver holds belief $p$ in the current period and state revelation does not happen this period. The value function satisfies the recursion $V_\delta(p) = \tilde{V}_\delta(p | p)$, where

$$\tilde{V}_\delta(\cdot | p) := \text{cav}_q[\mu(q - p) + \delta V_\delta(q) + (1 - \delta)(q \cdot \mu(1 - q) + (1 - q) \cdot \mu(-q))].$$

Ely (2017) studies an infinite-horizon information design problem whose value function also involves concavification. Unlike in Ely (2017), the current belief enters the objective function for our news-utility problem.

Our first result shows this recursion has a unique solution which increases in $\delta$ for any fixed $p \in [0, 1]$.

**Proposition 14.** For every $\delta \in [0, 1)$, the value function $V_\delta$ exists and is unique. Furthermore, $V_\delta(p)$ is increasing in $\delta$ for every $p \in [0, 1]$.

Figure 6 illustrates this result by plotting $V_\delta(p)$ for the quadratic news utility with $\alpha_p = 2$, $\alpha_n = 2.1$, $\beta_p = 1$, and $\beta_n = 0.2$ for three different values of $\delta : 0, 0.8, \text{and } 0.95$. (In fact, the monotonicity of the value function in $\delta$ also holds when there are more than two states.)

The monotonicity of $V_\delta$ in $\delta$ says that when the sender is benevolent and has commitment power, third-party leaks are harmful for the receiver’s expected welfare. This result can be explained intuitively as follows. Just as with increasing $T$ in the finite-horizon model, increasing $\delta$ expands the set of implementable belief paths. The idea behind implementing a payoff from a shorter horizon / lower $\delta$ is that the sender switches to babbling forever after certain histories. This switching happens at a deterministic calendar time in the finite-
Figure 6: The value function for $\delta = 0, 0.8, 0.95$. Consistent with Proposition 14, the value function is pointwise higher for higher $\delta$.

horizon setting but at a random time in the random-horizon setup, mimicking the random arrival of the state revelation period.

### 5.3 Gradual Good News Equilibria Without Commitment

Now we turn to equilibria of the random-horizon cheap talk game when the sender lacks commitment power. Analogously to the case of finite horizon, a *strict gradual good news* equilibrium (strict GGN) features a deterministic sequence of increasing posteriors $q^{(0)} < q^{(1)} < \ldots$ such that $q^{(0)} = \pi_0$ is the receiver’s prior before the game starts and $q^{(t)}$ is his belief in period $t$, provided state revelation has not occurred. An analog of Proposition 12 continues to hold.

**Proposition 15.** Let $P^*(\pi) \subseteq (\pi, 1]$ be those beliefs $p$ satisfying $N_B(p; \pi) = N_B(0; \pi)$. 
Suppose \( \mu \) exhibits diminishing sensitivity and loss aversion. There exists a gradual good news equilibrium with a (possibly infinite) sequence of intermediate beliefs \( q^{(1)} < q^{(2)} < \ldots \) if and only if \( q^{(j)} \in P^*(q^{(j-1)}) \) for every \( j = 1, 2, \ldots \), where \( q^{(0)} := \pi_0 \).

The \( P^* \) set is the same in the finite- and random-horizon environments. Corollary 6 then implies that even in the random-horizon environment where the game could continue for arbitrarily many periods, intermediate beliefs grow at an increasing rate in GGN equilibria for quadratic and square-roots \( \mu \), and there exists a finite bound on the number of periods of informative communication that applies for all \( \delta \in [0, 1) \).

6 Other Models of Preference over Non-Instrumental Information

6.1 Diminishing Sensitivity over News

The literature on reference-dependent preferences and news utility has focused on two-part linear gain-loss utility functions, which violate diminishing sensitivity. If \( \mu \) is two-part linear with loss aversion, then it follows from the martingale property of Bayesian beliefs that one-shot resolution is weakly optimal for the sender among all information structures. If there is strict loss aversion, then one-shot resolution does strictly better than any information structure that resolves uncertainty gradually. As our results have shown, more nuanced information structures emerge as optimal when the receiver exhibits diminishing sensitivity.

6.2 Anticipatory Utility

In our setup, a receiver who experiences anticipatory utility gets \( A(\sum \pi_t(\theta) \cdot v(\theta)) \) if she ends period \( t \) with posterior belief \( \pi_t \in \Delta(\Theta) \), where \( A: \mathbb{R} \to \mathbb{R} \) is a strictly increasing anticipatory-utility function. When \( A \) is the identity function (as in Kőszegi (2006)), the solution to the sender’s problem would be unchanged if we modified our model and let the receiver experience both anticipatory utility and news utility. This is because by the martingale property, the receiver’s ex-ante expected anticipatory utility in a given period is the same across all information structures. So, the ranking of information structures entirely depends on the news utility they generate. For a general \( A \), if the receiver only
experiences anticipatory utility, not news utility, then the sender has an optimal information structure that only releases information in \( t = 1 \), followed by uninformative babbling in all subsequent periods. For instance, Schweizer and Szech (2018) show that the best non-instrumental medical test for a patient with a concave anticipatory-utility function \( A \) is fully uninformative. The above argument establishes that even if the doctor can give the patient a series of tests on different days and even if \( A \) is not concave, the optimal test design involves a possibly informative test on the first day, followed by uninformative tests on all subsequent days. The rich dynamics of the optimal information structure in our news-utility model are thus absent in an anticipatory-utility model.

6.3 State-Dependent Suspense or Surprise Utility

A key distinction of our model from Ely, Frankel, and Kamenica (2015) is that changes in beliefs may bring utility or disutility to the receiver, depending on the nature of the news. By contrast, agents with suspense or surprise utilities always derive greater utility from larger movements in beliefs, regardless of the directions of these movements.

Ely, Frankel, and Kamenica (2015) also discuss state-dependent versions of suspense and surprise utilities, but this extension does not embed our model either. Suppose there are two states, \( \Theta = \{G, B\} \), and the agent has the suspense objective \( \sum_{t=0}^{T-1} u(\mathbb{E}_t(\sum_{\theta} \alpha_{\theta} \cdot (\pi_{t+1}(\theta) - \pi_t(\theta))^2)) \) or the surprise objective \( \sum_{t=1}^{T} u(\sum_{\theta} \alpha_{\theta} \cdot (\pi_t(\theta) - \pi_{t-1}(\theta))^2) \), where \( \alpha_G, \alpha_B > 0 \) are state-dependent scaling weights. We must have \( \pi_{t+1}(G) - \pi_t(G) = -(\pi_{t+1}(B) - \pi_t(B)) \), so path-wise \( (\pi_{t+1}(G) - \pi_t(G))^2 = (\pi_{t+1}(B) - \pi_t(B))^2 \). This shows that the new objectives obtained by applying two possibly different scaling weights \( \alpha_G \neq \alpha_B \) to states \( G \) and \( B \) are identical to the ones that would be obtained by applying the same scaling weight \( \alpha = \frac{\alpha_G + \alpha_B}{2} \) to both states. Due to this symmetry in preference, the optimal information structure for entertaining an agent with state-dependent suspense or surprise utility does not treat the two states asymmetrically, in contrast to a central prediction of diminishing sensitivity in our model.

7 Concluding Discussion

In this work, we have studied how an informed sender optimally communicates with a receiver who derives diminishing gain-loss utility from changes in beliefs. If we think that diminishing sensitivity to the magnitude of news is psychologically realistic in this domain, then the stark
predictions of the ubiquitous two-part linear models may be misleading. In the presence of diminishing sensitivity, richer information structures emerge as optimal for the committed sender. For example, the optimal information structure can feature asymmetric treatments of good and bad news. If the sender lacks commitment power, diminishing sensitivity leads to novel credibility problems that inhibit any meaningful communication when the receiver has no loss aversion.

Some of our predictions can empirically distinguish news utility with diminishing sensitivity from other models of belief-based preference over non-instrumental information, including the two-part linear news-utility model. Proposition 7, for example, suggests a laboratory experiment where a sequence of binary events determines whether a baseline state or an alternative state realizes, with the alternative state happening only if all of the binary events are “successful.” Consider two treatments that have the same success probabilities for the binary events, but differ in terms of whether subjects get a higher consumption or a lower consumption in the alternative state compared with the baseline state. Diminishing sensitivity over news predicts that more subjects should prefer one-shot resolution when consumption is lower in the alternative state than when it is higher in the alternative state, a hypothesis we plan to test in future work.

References


Appendix

A Proofs

In the proofs, we will often use the following fact about news-utility functions with diminishing sensitivity. We omit its simple proof.

Fact 1. Let $d_1, d_2 > 0$ and suppose $\mu(0) = 0$.

- (sub-additivity in gains) If $\mu''(x) < 0$ for all $x > 0$, then $\mu(d_1 + d_2) < \mu(d_1) + \mu(d_2)$.
- (super-additivity in losses) If $\mu''(x) > 0$ for all $x < 0$, then $\mu(-d_1 - d_2) > \mu(-d_1) + \mu(-d_2)$.

A.1 Proof of Proposition 1

Proof. We first justify by backwards induction that the value function is indeed given by

$$U_t^*(x) = (\text{cav}U_t(\cdot | x))(x),$$

for all $x \in \Delta(\Theta)$ and all $t \leq T - 1$, and that it is continuous in $x$.

If the receiver enters period $t = T - 1$ with the belief $x \in \Delta(\Theta)$, the sender faces the following maximization problem.

$$(P_{T-1}) \max_{\mu \in \Delta(\Delta(\Theta)), \mathbb{E}[\mu] = x} \int_{\Delta(\Theta)} U_{T-1}(p | x) d\mu(p).$$

This is because any sender strategy $\sigma_{T-1}$ induces a Bayes plausible distribution of posterior beliefs, $\mu$ with $\mathbb{E}[\mu] = x$, and conversely every such distribution can be generated by some sender strategy, as in Kamenica and Gentzkow (2011). It is well-known that the value of problem $P_{T-1}$ is $(\text{cav}U_{T-1}(\cdot | x))(x)$, justifying $U_{T-1}^*(x)$ as the value function for any $x \in \Delta(\Theta)$. The objective in $P_{T-1}$ is continuous in $p$ (by assumption on $N$) and hence in $\mu$, and furthermore the constraint set $\{\mu \in \Delta(\Delta(\Theta)) : \mathbb{E}[\mu] = x\}$ is continuous in $x$. Therefore, $x \mapsto U_{T-1}^*(x)$ is continuous by Berge’s Maximum Theorem.

Assume that we have shown that value function is continuous and given by $U_t^*(x)$ for all $t \geq S$. If the receiver enters period $t = S - 1$ with belief $x$, then the sender’s value must be:
(P_t) \max_{\mu \in \Delta(\Delta(\Theta)), \mathbb{E}[\mu] = x} \int_{\Delta(\Theta)} N(p \mid x) + U_{t+1}^*(p) d\mu(p)

using the inductive hypothesis that $U_{t+1}^*(p)$ is the period $t + 1$ value function. But $N(p \mid x) + U_{t+1}^*(p) = U_t(p \mid x)$ by definition, and it is continuous by the inductive hypothesis. So by the same arguments as in the base case, $U_{S-1}^*(x)$ is the time-$(S-1)$ value function and it is continuous, completing the inductive step.

In the first period, by Carathéodory’s theorem, there exist weights $w_1, \ldots, w_K \geq 0$, beliefs $q_1, \ldots, q_K \in \Delta(\Theta)$, with $\sum_{k=1}^K w_k = 1$, $\sum_{k=1}^K w_k q_k = x$, such that $U_1^*(\pi_0) = \sum_{k=1}^K w_k U_1(q_k \mid \pi_0)$. Having now shown $U_2^*$ is the period-2 value function, there must exist an optimal information structure where $\sigma_1(\cdot \mid \theta)$ induces beliefs $q_k$ with probability $w_k$. This information structure induces one of the beliefs $q_1, \ldots, q_K$ in the second period. Repeating the same procedure for subsequent periods establishes the proposition.

\[ A.2 \text{ Proof of Proposition 2} \]

**Proof.** Suppose $T = 2$. Consider the following family of information structures, indexed by $\epsilon > 0$. Order the states based on $E_c \sim F_\theta[v(c)]$ and label them $\theta_L, \theta_2, \ldots, \theta_{K-1}, \theta_H$. Let $M = \{m_L, m_2, \ldots, m_{K-1}, m_H\}$. Let $\sigma_1(\theta_k)(m_k) = 1$ for $2 \leq k \leq K-1$, $\sigma_1(\theta_H)(m_H) = 1$, and $\sigma_1(\theta_L)(m_L) = x$, $\sigma_1(\theta_L)(m_H) = 1 - x$ for some $x \in (0,1)$ so that the posterior belief after observing $m_H$ is $(1 - \epsilon)1_H \oplus \epsilon 1_L$.

For every $\epsilon > 0$, the information structure just described leads to one-shot resolution of states $\theta \notin \{\theta_L, \theta_H\}$. The difference between its expected news utility and that of one-shot resolution is $W(\epsilon)$, given by

\[
\pi_0(\theta_H) \cdot [N((1 - \epsilon)1_H \oplus \epsilon 1_L \mid \pi_0) + N(1_H \mid (1 - \epsilon)1_H \oplus \epsilon 1_L) - N(1_H \mid \pi_0)] \\
+ \frac{\epsilon}{1 - \epsilon} \pi_0(\theta_H) \cdot [N((1 - \epsilon)1_H \oplus \epsilon 1_L \mid \pi_0) + N(1_L \mid (1 - \epsilon)1_H \oplus \epsilon 1_L) - N(1_L \mid \pi_0)].
\]

$W$ is continuously differentiable away from 0 and $W(0) = 0$. To show that $W(\epsilon) > 0$ for some $\epsilon > 0$, it suffices that $\lim_{\epsilon \to 0} W'(\epsilon) > 0$. Using the continuous differentiability of $N$
except when its two arguments are identical, this limit is
\[
\lim_{\epsilon \to 0^+} \frac{N((1 - \epsilon)1_H \oplus \epsilon 1_L \mid \pi_0) - N(1_H \mid \pi_0)}{\epsilon} + \lim_{\epsilon \to 0^+} \frac{N(1_H \mid (1 - \epsilon)1_H \oplus \epsilon 1_L)}{\epsilon}
\]
\[+ N(1_H \mid \pi_0) + N(1_L \mid 1_H) - N(1_L \mid \pi_0).\]

Simple rearrangement gives the expression from Proposition 2. The expression for the case of mean-based \( \mu \) follows by algebra, noting that \( N((1 - x)1_H \oplus x1_L \mid \pi_0) = \mu((1 - x) - v_0) \) for \( x \in [0, 1] \).

If \( T > 2 \), then note the sender’s \( T \)-period problem starting with prior \( \pi_0 \) has a value at least as large as the 2-period problem with the same prior. On the other hand, one-shot resolution brings the same total expected news utility regardless of \( T \).

A.3 Proof of Corollary 1

Proof. We verify Proposition 2’s condition
\[
\mu'(0^+) + \mu(1 - \pi_0) - \mu(-\pi_0) > -\mu(-1) + \mu'(1 - \pi_0).
\]

We have that
\[
LHS = \alpha_p + \alpha_p(1 - \pi_0) - \beta_p(1 - \pi_0)^2 - [\beta_n \pi_0^2 - \alpha_n \pi_0]
\]
\[
RHS = [-\beta_n + \alpha_n] + [\alpha_p - 2\beta_p(1 - \pi_0)]
\]

By algebra,
\[
LHS - RHS = (1 - \pi_0)(\alpha_p - \alpha_n) + (1 - \pi_0^2)(\beta_p + \beta_n).
\]

Given that \( (\alpha_n - \alpha_p) \leq (\beta_p + \beta_n) \) and \( 1 - \pi_0^2 > 1 - \pi_0 \) for \( 0 < \pi_0 < 1 \),
\[
LHS - RHS > -(1 - \pi_0^2)(\beta_p + \beta_n) + (1 - \pi_0^2)(\beta_p + \beta_n) = 0.
\]

A.4 Proof of Corollary 2

Proof. This follows from Proposition 2 because \( \mu'(0^+) = \infty \) for the power function.
Figure A.1: New belief about consumption after the muddled message $m_*$ in an environment with 4 states, compared with the old belief given by the prior $\pi_0$.

### A.5 Proof of Proposition 3

**Proof.** Suppose $\Theta = \{\theta_1, ..., \theta_K\}$ and assume without loss the states are associated with consumption levels $c_1 < ... < c_K$.

Let the message space be $M = \{m_1, ..., m_K, m_*\}$. In the first period,

- $\sigma_1(m_k \mid \theta_k) = 1$ for $1 \leq k \leq K - 2$,
- $\sigma_1(m_* \mid \theta_{K-1}) = 1$,
- $\sigma_1(m_* \mid \theta_K) = \frac{\pi_0(\theta_{K-1})}{1 - \pi_0(\theta_K)}$,
- $\sigma_1(m_K \mid \theta_K) = 1 - \sigma_1(m_* \mid \theta_K)$.

So, message $m_k$ perfectly reveals state $\theta_k$, whereas $m_*$ is a “muddled” message that implies the state is either $\theta_{K-1}$ or $\theta_K$. By simple algebra, the probability that the receiver assigns to state $\theta_K$ after $m_*$ is the same as the prior belief,

$$P[\theta_K \mid m_*] = \frac{\pi_0(\theta_K) \cdot \sigma_1(m_* \mid \theta_K)}{\pi_0(\theta_K) \cdot \sigma_1(m_* \mid \theta_K) + \pi_0(\theta_{K-1}) \cdot 1} = \pi_0(\theta_K).$$

In the second period, the information structure perfectly reveals the true state regardless of the last message, $\sigma_2(m_k \mid \theta_k) = 1$ for all $1 \leq k \leq K$.

To compute the news utility of the muddled message $m_*$, note that at percentiles $p \in [0, \pi_0(\theta_1))$, the change in $p$-percentile consumption utility is $v(c_{K-1}) - v(c_1)$. Similarly, for $2 \leq k \leq K - 2$, the change in consumption utility at percentile $p \in \left[\sum_{j=1}^{k-2} \pi_0(\theta_j), \pi_0(\theta_k) + \sum_{j=1}^{k-1} \pi_0(\theta_j)\right]$ is $v(c_{K-1}) - v(c_k)$. There are no changes at percentiles above $\sum_{j=1}^{K-2} \pi_0(\theta_j)$. 

40
If \( \theta = \theta_{K-1} \), total news utility from receiving \( m_* \) then \( m_{K-1} \) is

\[
\left[ \sum_{k=1}^{K-2} \pi_0(\theta_k) \cdot \mu(v(c_{K-1}) - v(c_k)) \right] + \pi_0(\theta_K) \cdot \mu(v(c_{K-1}) - v(c_K)).
\]

This is identical to the news utility from one-shot resolution in state \( \theta_{K-1} \). Similarly, the information structure just constructed gives the same news utility as one-shot resolution when the state is \( \theta_k \) for \( 1 \leq k \leq K - 2 \), and when the state is \( \theta_K \) and the receiver gets \( m_K \) in period 1.

When the receiver sees \( m_* \) in period 1 and \( m_K \) in period 2 in state \( \theta_K \), an event that happens with strictly positive probability since \( \pi_0(\theta_{K-1}) < 1 - \pi_0(\theta_K) \) as \( K \geq 3 \), he gets strictly more news utility than from one-shot resolution.

If \( \theta = \theta_K \), total news utility from receiving \( m_* \) then \( m_K \) is

\[
\left[ \sum_{k=1}^{K-2} \pi_0(\theta_k) \cdot \mu(v(c_{K-1}) - v(c_k)) \right] + \left[ \sum_{k=1}^{K-1} \pi_0(\theta_k) \cdot \mu(v(c_K) - v(c_{K-1})) \right],
\]

while one-shot resolution gives

\[
\sum_{k=1}^{K-1} \pi_0(\theta_k) \cdot \mu(v(c_K) - v(c_k)).
\]

For each \( 1 \leq k \leq K - 2 \) (non-empty since \( K \geq 3 \)),

\[
\mu(v(c_K) - v(c_{K-1})) + \mu(v(c_{K-1}) - v(c_k)) > \mu(v(c_K) - v(c_k))
\]

by sub-additivity in gains. This shows the constructed information structure gives strictly more news utility.

**A.6 Proof of Proposition 5**

*Proof.* Let (1) be the following geometric condition: the concavification of \( U_1(p|\pi_0) \) involves a linear segment starting at the pair \( p = 0, U_1(0|\pi_0) \) which is strictly above \( U_1(\pi_0|\pi_0) \) when evaluated at \( p = \pi_0 \). We need to show that (1) holds true if and only if partial bad news
are suboptimal. It is clear that whenever the geometric condition (!) is satisfied, partial bad news are suboptimal as the posterior induced in the bad state must be equal to 0 with probability one. On the other hand, knowing that the posterior induced in the bad state is 0 with probability 1 implies two possibilities: (i) either perfect revelation of the state is optimal, or (ii) the optimal information structure involves partial good news and perfect revelation of the bad state. In either case, the only posterior induced in the bad state is that of 0, i.e. the concavification has to include the point \((0, U_1(0|\pi_0))\). From the definition of concavification and the fact that it is supported on two points of the graph of \(q \to U_1(q|\pi_0)\), it follows that the concavification has to include a linear segment starting at \((0, U_1(0|\pi_0))\), thus (!) should hold true.

Because of the two-point support feature of the concavification and the fact that the average of the posteriors needs to be equal to the prior \(\pi_0 \in (0, 1)\), this implies that there is a \(q > \pi_0\), which is the second point of support for the concavification and the linear segment. In case (i) above, it holds \(q = 1\) whereas in case (ii) it holds \(q < 1\).

A.7 Proof of Corollary 3

We first prove a sufficient condition for the sub-optimality of information structures with partial bad news with \(T = 2\). Consider the chord connecting \((0, U_1(0 | \pi_0))\) and \((\pi_0, U_1(\pi_0 | \pi_0))\) and let \(\ell(x)\) be its height at \(x \in [0, \pi_0]\). Let \(D(x) := \ell(x) - U_1(x | \pi_0)\).

**Lemma A.1.** For this chord to lie strictly above \(U_1(p | \pi_0)\) for all \(p \in (0, \pi_0)\), it suffices that \(D'(0) > 0\), \(D'(\pi_0) < 0\), and \(D''(p) = 0\) for at most one \(p \in (0, \pi_0)\).

**Proof.** We need \(D > 0\) in the region \((0, \pi_0)\). We know that \(D(0) = D(\pi_0) = 0\). Given the conditions in the statement and the twice-differentiability of \(D\) in \((0, \pi_0)\) it follows that \(D''\) changes sign only once. Moreover, it also follows that \(D > 0\) in a right-neighborhood of \(x = 0\) and a left-neighborhood of \(x = \pi_0\). Suppose \(D\) has an interior minimum at \(x_0 \in (0, \pi_0)\). Then it holds \(D''(x_0) \geq 0\).

Suppose \(D''(x) > 0\) for all small \(x\). Then it follows \(x_0 \leq p\), where we set \(p = \pi_0\) if \(p\) doesn’t exist. Because \(D''(x) \geq 0\) for all \(x \leq p\) we have that \(D'(x) > 0\) for all \(x \leq p\). In particular also \(D(x) > 0\) for all such \(x\) due to the Fundamental Theorem of Calculus. Thus, the interior minimum is positive and so the claim about \(D\) in \((0, \pi)\) is proven in this case.
Suppose instead that \( D''(x) < 0 \) for all \( x \) near enough to 0. Then it follows that \( x_0 \geq p \). In particular, for all \( x > p \) we have \( D''(x) > 0 \). Since the derivative is strictly increasing for all \( x \in (0, \pi_0) \) and \( D'(\pi_0) < 0 \) we have that \( D'(x) < 0 \) for all \( x \in (x_0, \pi_0) \). In particular, from the Fundamental Theorem of Calculus, \( D(\pi_0) \) is strictly below \( D(x_0) \). Since \( D(\pi_0) = 0 \) we have again that \( D(x_0) > 0 \).

Given the boundary values of \( D \) and the signs of the derivatives at 0, \( \pi_0 \) and that any interior minimum of \( D \) is strictly positive, we have covered all cases and so shown that \( D > 0 \) in \((0, \pi_0)\).

Now we verify that the condition in Lemma A.1 holds for the quadratic news utility, which in turn verifies the condition of Proposition 5 for \( q = \pi_0 \) and shows partial bad news information structures to be strictly suboptimal.

**Proof.** Clearly, \( D(p) \) is a third-order polynomial, so \( D''(p) \) has at most one root.

For \( p < \pi_0 \), we have the derivative

\[
\frac{d}{dp} U(p \mid \pi_0) = 2\beta_n(p - \pi_0) + \alpha_n + \alpha_p(1 - p) - \beta_p(1 - p)^2 \\
+ p(-\alpha_p + 2\beta_p(1 - p)) - (\beta_n p^2 - \alpha_n p) + (1 - p)(2\beta_n p - \alpha_n)
\]

The slope of the chord between 0 and \( \pi_0 \) is: \( \alpha_p - \beta_p + (2\beta_p - \alpha_p + \alpha_n)\pi_0 - (\beta_p + \beta_n)\pi_0^2 \). So, after straightforward algebra, \( D'(0) = (2(\beta_p + \beta_n) - (\alpha_p - \alpha_n))\pi_0 - (\beta_p + \beta_n)\pi_0^2 \). Applying weak loss aversion with \( z = 1 \), \( \alpha_p - \alpha_n \leq \beta_p - \beta_n \). This shows

\[
D'(0) \geq (2(\beta_p + \beta_n) - (\beta_p - \beta_n))\pi_0 - (\beta_p + \beta_n)\pi_0^2 \\
= (\beta_p + \beta_n)\pi_0(1 - \pi_0) + 2\beta_n\pi_0 > 0
\]

for \( 0 < \pi_0 < 1 \).

We also derive \( D'(\pi_0) = (\alpha_p - 2\beta_p - 2\beta_n - \alpha_n)\pi_0 + (2\beta_p + 2\beta_n)\pi_0^2 \). Note that this is a convex parabola in \( \pi_0 \), with a root at 0. Also, the parabola evaluated at 1 is equal to \( \alpha_p - \alpha_n \leq 0 \), where the inequality comes from the weak loss aversion with \( z = 0 \). This implies \( D'(\pi_0) < 0 \) for \( 0 < \pi_0 < 1 \). 

\[\square\]
A.8 Proof of Proposition 4

Proof. We show that one-shot resolution gives weakly higher news utility conditional on each state, and strictly higher news utility conditional on at least one \( \theta \in \Theta_B \).

When \( \theta \in \Theta_B \), \( \mathbb{P}_{(M,\sigma)} \)-almost surely the expectations in different periods form a decreasing sequence \( v_0 \geq v_1 \geq ... \geq v_T = v(c_\theta) \). By super-additivity in losses, \( \sum_{t=1}^{T} \mu(v_t - v_{t-1}) \leq \mu(v_T - v_0) = \mu(v(c_\theta) - v_0) \). This shows \( \mathbb{P}_{(M,\sigma)} \)-almost surely the ex-post news utility in state \( \theta \) is no larger than \( \mu(v(c_\theta) - v_0) \), the news utility from one-shot resolution.

Let \( E \) be the event where the receiver’s expectation strictly decreases two or more times. From the definition of strict gradual bad news, there exists some \( \theta^* \in \Theta_B \) so that \( \mathbb{P}_{(M,\sigma)}[E | \theta^*] > 0 \). On \( E \cap \{ \theta^* \} \), \( \sum_{t=1}^{T} \mu(v_t - v_{t-1}) < \mu(v(c_{\theta^*}) - v_0) \) from super-additivity in losses, which means the expected news utility conditional on \( E \cap \{ \theta^* \} \) is strictly lower than that of one-shot resolution. Combined with the fact that the ex-post news utility in state \( \theta^* \) is always weakly lower than \( \mu(v(c_{\theta^*}) - v_0) \), this shows expected news utility in state \( \theta^* \) is strictly lower than that of one-shot resolution.

Conditional on any state \( \theta \in \Theta_G \), there is some random period \( t^* \in \{0,...,T-1\} \) so that \( v_t \) is weakly decreasing up to \( t = t^* \) and \( v_t = v(c_\theta) \) for \( t > t^* \). If \( t^* = 0 \), then this belief path yields the same news utility as one-shot resolution. If \( t^* \geq 1 \), then the total news utility is

\[
\sum_{t=1}^{t^*} \mu(v_t - v_{t-1}) + \mu(v(c_\theta) - v_{t^*}).
\]

By sub-additivity in gains,

\[
\sum_{t=1}^{t^*} \mu(v_t - v_{t-1}) \leq \mu(v_{t^*} - v_0);
\]

and for the same reason,

\[
\mu(v(c_\theta) - v_{t^*}) \leq \mu(v_0 - v_{t^*}) + \mu(v(c_\theta) - v_0)
\]

as we must have \( v_{t^*} \leq v_0 \). Total news utility is therefore bounded above by

\[
\mu(v_{t^*} - v_0) + \mu(v_0 - v_{t^*}) + \mu(v(c_\theta) - v_0).
\]

By weak loss aversion, \( \mu(v_{t^*} - v_0) + \mu(v_0 - v_{t^*}) \leq 0 \), therefore total news utility is no larger
than that of one-shot resolution, \( \mu(v(c_0) - \pi_0) \).

A.9 Proof of Proposition 6

**Proof.** We have

\[
\frac{d}{dp} U(p \mid \pi_0) = 2\alpha_p - \alpha_n - \beta_p + 2\beta_p \pi_0 + p(-2\alpha_p + 2\beta_p + 2\alpha_n + 2\beta_n) + p^2(-3\beta_p - 3\beta_n)
\]

Further, \( p \) times slope of chord is:

\[
U(p \mid \pi_0) - U(0 \mid \pi_0) = U(p \mid \pi_0) - (\beta_n \pi_0^2 - \alpha_n \pi_0)
= \pi_0(-\alpha_p + \alpha_n) + \pi_0^2(-\beta_p - \beta_n) + p(2\alpha_p - \alpha_n - \beta_p)
+ p^2(-\alpha_p + \beta_p + \alpha_n + \beta_n) + p^3(-\beta_p - \beta_n) + p\pi_0(2\beta_p)
\]

Equating \( p \cdot \frac{d}{dp} U(p \mid \pi_0) = U(p \mid \pi_0) - U(0 \mid \pi_0) \), we get

\[
\pi_0(\alpha_n - \alpha_p) - (\beta_p + \beta_n)\pi_0^2 = p^2(\alpha_n - \alpha_p + \beta_n + \beta_p) - p^3(2\beta_p + 2\beta_n).
\]

Define \( c = \frac{\alpha_n - \alpha_p}{\beta_n + \beta_p} \). Then, we can write the implicit function as

\[
\pi_0 c - \pi_0^2 = p^2(1 + c) - 2p^3.
\]

That for every \( 0 \leq c \leq 1 \) and \( \pi_0 \in (0, 1) \) this has a solution we know it from the fact that the chord condition is always satisfied for quadratic specification. We want to take derivatives though and maybe try and solve explicitly for the function \( p(\pi_0, c) \).

The condition to apply implicit function theorem: define the function \( f(\pi_0, p, c) = p^2(1 + c) - 2p^3 - \pi_0 c + \pi_0^2 \) with domain \((0, 1)^3\); then we need \( \partial_p f(\pi_0, p, c) \neq 0 \). If this is true, then we can solve locally for \( p(\pi_0, c) \) and then also calculate the local derivative/comparative statics we do below locally. We note that \( \partial_p f(\pi_0, p, c) = 2p(1 + c) - 6p^2 \). Thus, the only constellation where this would be zero is if \( p = \frac{1+c}{3} =: \hat{p} \in \left[ \frac{1}{3}, \frac{2}{3} \right] \), given the sufficient condition \( c \in (0, 1) \) that we are imposing (see Corollary 1) but we leave out the boundary values for a second. Now, let us see for a fixed \( c \), what \( \pi_0 \) would give \( \hat{p} \) (because we only focus on region where implicit function gives out a solution). This would mean solving for \( \pi_0 \) the quadratic
equation
\[ \pi_0^2 - \pi_0 c + \frac{1}{27}(1 + c)^3 = 0. \]  (1)

Let’s calculate the discriminant as a function of \( c \). It is given as \( D(c) = c^2 - \frac{4}{27}(1 + c)^3 \). Note that \( D'(c) = \frac{2}{9}(2 - c)(2c - 1) \). In particular, \( D \) is falling from \( c = 0 \) till \( c = \frac{1}{2} \) and increasing from then on till \( c = 1 \). We note also that \( D(0) < 0, D(1) < 0 \) so that overall it follows that \( D(c) < 0 \) for all \( c \in [0, 1] \). In particular, it holds that Equation (1) is never solvable! This means that \( \partial_p f \) never changes sign in \( (0, 1)^3 \cap \{(p, \pi_0, c) : \pi_0 c - \pi_0^2 = p^2(1 + c) - 2p^3\} \) (\( f \) is a smooth function on its domain). Thus, implicit function theorem is applicable for all \((\pi_0, c) \in (0, 1)^2\).

Totally differentiating, we get:

\[
d\pi_0 \cdot (\alpha_n - \alpha_p) - (\beta_p + \beta_n) 2\pi_0 \cdot d\pi_0 = 2p \cdot dp \cdot (\alpha_n - \alpha_p + \beta_n + \beta_p) - 3p^2 \cdot dp \cdot (2\beta_p + 2\beta_n),
\]

which can be rearranged to \( \frac{dp}{d\pi_0} = \frac{c - 2\pi_0}{2p^2(1 + c) - 6p^3} \). We note that we showed above the denominator of this expression never changes sign. Given that we know it’s negative at \( c = 0 \) we conclude that it’s always negative for all \( c \) and all \( \pi_0 \in (0, 1) \). It follows that unless \( c = 0 \), \( p(\pi_0) \) is falling till some prior and increasing afterwards. For \( c = 0 \) it is strictly increasing all the way. Note that an implication of the shape for the case of \( c > 0 \) is that \( p(0, c) = \frac{1+ c}{2} \) (because the other root which is zero would lead to a contradiction of the shape, given that \( p \in [0, 1] \)). Thus, the amount of partial good news in the good state remains bounded away from zero as the prior indicates more and more that overall the state is bad with high probability. □

### A.10 Proof of Proposition 7

**Proof.** Consider an agent who prefers B over A. In state A, he gets \( \mu(-\rho_0) \) with one-shot information, but \( \sum_{t=1}^T \mu(\rho_t - \rho_{t-1}) \) with gradual information. For each \( t, \rho_t - \rho_{t-1} < 0 \), and furthermore \( \sum_{t=1}^T \rho_t - \rho_{t-1} = -\rho_0 \) by telescoping and using the fact that \( \rho_T = 0 \). Due to super-additivity in losses, we get that \( \mu(-\rho_0) > \sum_{t=1}^T \mu(\rho_t - \rho_{t-1}) \). In state B, he gets \( \mu(1 - \rho_0) \) with one-shot information. With gradual information, let \( T \leq \hat{T} \) be the first period where the coin toss comes up tails. His news utility is \( \left[ \sum_{t=1}^{\hat{T}-1} \mu(\rho_t - \rho_{t-1}) \right] + \mu(1 - \rho_{\hat{T}-1}) \) where each \( \rho_t - \rho_{t-1} < 0 \) for \( 1 \leq t \leq \hat{T} - 1 \). Again by super-additivity in losses, \( \sum_{t=1}^{\hat{T}-1} \mu(\rho_t - \rho_{t-1}) < \mu(\rho_{\hat{T}-1} - \rho_0) \). By
sub-additivity in gains, \( \mu(1 - \rho_{T-1}) < \mu(\rho_0 - \rho_{T-1}) + \mu(1 - \rho_0) \leq -\mu(\rho_{T-1} - \rho_0) + \mu(1 - \rho_0) \), where the weak inequality follows since \( \lambda \geq 1 \). Putting these pieces together,

\[
\sum_{t=1}^{\hat{T}-1} \mu(\rho_t - \rho_{t-1}) + \mu(1 - \rho_{T-1}) < \mu(\rho_{T-1} - \rho_0) - \mu(\rho_{T-1} - \rho_0) + \mu(1 - \rho_0) = \mu(1 - \rho_0)
\]

as desired.

Now consider an agent who prefers A over B. We show that when \( \lambda = 1 \), the agent strictly prefers gradual information to one-shot information. By continuity of news utility in \( \lambda \), the same strict preference must also hold for \( \lambda \) in an open neighborhood around 1.

In state A, the agent gets \( \mu(1 - \pi_0) \) with one-shot information, but \( \sum_{t=1}^{T} \mu(\pi_t - \pi_{t-1}) \) with gradual information. For each \( t \), \( \pi_t - \pi_{t-1} > 0 \), and furthermore \( \sum_{t=1}^{T} \pi_t - \pi_{t-1} = 1 - \pi_0 \) by telescoping and using the fact that \( \pi_T = 1 \). Due to sub-additivity in gains, we get that \( \sum_{t=1}^{T} \mu(\pi_t - \pi_{t-1}) > \mu(1 - \pi_0) \). In state B, he gets \( \mu(-\pi_0) \) with one-shot information. With gradual information, let \( \hat{T} \leq T \) be the first period where the \( X_{\hat{T}} = 0 \). His news utility is \( \sum_{t=1}^{\hat{T}-1} \mu(\pi_t - \pi_{t-1}) + \mu(-\pi_{\hat{T}-1}) \) where each \( \pi_t - \pi_{t-1} > 0 \) for \( 1 \leq t \leq \hat{T} - 1 \). Again by sub-additivity in gains, \( \sum_{t=1}^{\hat{T}-1} \mu(\pi_t - \pi_{t-1}) > \mu(\pi_{\hat{T}-1} - \pi_0) \). By super-additivity in losses, \( \mu(-\pi_{\hat{T}-1}) > \mu(-\pi_{\hat{T}-1}) + \mu(-\pi_0) = -\mu(\pi_{\hat{T}-1} - \pi_0) + \mu(-\pi_0) \), where the equality comes from the fact that \( \lambda = 1 \) so \( \mu \) is symmetric about 0. Putting these pieces together,

\[
\sum_{t=1}^{\hat{T}-1} \mu(\pi_t - \pi_{t-1}) + \mu(-\pi_{\hat{T}-1}) > \mu(\pi_{\hat{T}-1} - \pi_0) - \mu(\pi_{\hat{T}-1} - \pi_0) + \mu(-\pi_0) = \mu(-\pi_0)
\]

as desired. \( \square \)

### A.11 Proof of Proposition 8

**Proof.** (1) Suppose \( \mu \) is two-part linear with \( \mu(x) = kx \) for \( x \geq 0 \), \( \mu(x) = \lambda kx \) for \( x < 0 \), where \( k > 0, \lambda \geq 1 \). Then, agents preferring either state will strictly prefer one-shot information over gradual information. Indeed, since news utility is proportional to negative of expected movement in beliefs, and since \( \mathbb{E}[\sum_{t=1}^{T} \pi_t - \pi_{t-1}] = \mathbb{E}[\sum_{t=1}^{T} |(1 - \rho_t) - (1 - \rho_{t-1})|] = \mathbb{E}[\sum_{t=1}^{T} |\rho_t - \rho_{t-1}|] \), agents preferring state A and state B also derive the same amount of news
utility from each informational structure and hence have the same intensity of preference for one-shot information.

If \( \lambda = 1 \), agents do not exhibit strict preference for either information structure.

(2) Anticipatory utility. If \( u \) is linear, then agents are indifferent between gradual and one-shot information, so (up to tie-breaking) the agents preferring states A and B have the same preference over information structure. If \( u \) is strictly concave, then for \( 1 \leq t \leq T - 1 \),

\[
E[u(\pi_t)] < u(\pi_0) \quad \text{and} \quad E[u(\rho_t)] < u(\rho_0)
\]

by combining the martingale property and Jensen’s inequality. So all agents strictly prefer to keep their prior beliefs until the last period and will therefore all choose one-shot information.

(3) Suspense and surprise. Ely, Frankel, and Kamenica (2015) mention a “state-dependent” specification of their surprise and suspense utility functions. With two states, A and B, their specification uses weights \( \alpha_A, \alpha_B > 0 \) to differentially re-scale belief-based utilities for movements in the two different directions. Specifically, their re-scaled suspense utility is

\[
\sum_{t=0}^{T-1} u \left( E_t \left[ \alpha_A \cdot (\pi_{t+1} - \pi_t)^2 + \alpha_B \cdot (\rho_{t+1} - \rho_t)^2 \right] \right)
\]

and their re-scaled surprise utility is

\[
E \left[ \sum_{t=1}^{T} u \left( \alpha_A \cdot (\pi_{t+1} - \pi_t)^2 + \alpha_B \cdot (\rho_{t+1} - \rho_t)^2 \right) \right].
\]

We may consider agents with opposite preferences over states A and B as agents with different pairs of scaling weights \( (\alpha_A, \alpha_B) \). Specifically, say there are \( \alpha_{\text{High}} > \alpha_{\text{Low}} > 0 \). For an agent preferring A, \( \alpha_A = \alpha_{\text{High}}, \alpha_B = \alpha_{\text{Low}} \). For an agent preferring B, \( \alpha_A = \alpha_{\text{Low}}, \alpha_B = \alpha_{\text{High}} \). But note that we always have \( \pi_{t+1} - \pi_t = -(\rho_{t+1} - \rho_t) \), so along every realized path of beliefs, \( (\pi_{t+1} - \pi_t)^2 = (\rho_{t+1} - \rho_t)^2 \). This means these two agents with the opposite scaling weights actually have identical objectives and therefore will have the same preference over gradual or one-shot information.

A.12 Proof of Lemma 1

Proof. Part 1. Fix a prior \( \pi_0 \) and a pair \( (\tilde{M}, \tilde{\sigma}) \) which induces an equilibrium as in Definition 3. We focus on the case that \( |\tilde{M}| > 2 \) as the other cases are trivial.

Let \( M = \{g, b\} \) and we will inductively define the sender’s strategy \( \sigma_t \) on \( t \) so that \((M, \sigma)\)
is another equilibrium which delivers the same expected utility as \((\bar{M}, \bar{\sigma})\). In doing so we will successively define a sequence of subsets of histories, \(H_{\text{int}}^t \subseteq M^t\) and \(\bar{H}_{\text{int}}^t \subseteq \bar{M}^t\), which are length \(t\) histories associated with interior equilibrium beliefs about the state in the new and old equilibria, as well as a map \(\phi\) that associates new histories to old ones.

Let \(H_{\text{int}}^0 = \bar{H}_{\text{int}}^0 := \{\emptyset\}\), \(\phi(\emptyset) = \emptyset\).

Once we have defined \(\sigma_{t-1}, H_{\text{int}}^t, \bar{H}_{\text{int}}^t\) and \(\phi : H_{\text{int}}^t \rightarrow \bar{H}_{\text{int}}^t\), we then define \(\sigma_t\). If \(h^t \notin \bar{H}_{\text{int}}^t\), then simply let \(\sigma_t(h^t, \theta)(g) = 0.5\) for both \(\theta \in \{G, B\}\). For each \(h^t \in H_{\text{int}}^t\), by the definition of \(\bar{H}_{\text{int}}^t\), the equilibrium belief \(\pi_{t-1}\) associated with \(\phi(h^t)\) in the old equilibrium satisfies \(0 < \pi_{t-1} < 1\). Let \(\Phi_G(h^t-1)\) and \(\Phi_B(h^t-1)\) represent the sets of posterior beliefs that the sender induces with positive probability in the good and bad states following public history \(\phi(h^t-1) \in \bar{H}_{\text{int}}^t\) in \((\bar{M}, \bar{\sigma})\).

We must have \(\Phi_G(h^t-1) \setminus \Phi_B(h^t-1) \subseteq \{1\}\) and \(\Phi_B(h^t-1) \setminus \Phi_G(h^t-1) \subseteq \{0\}\), since any message unique to either state is conclusive news of the state. We construct \(\sigma_t(h^t-1, \theta)\) based on the following four cases.

Case 1: \(1 \in \Phi_G(h^t-1)\) and \(0 \notin \Phi_B(h^t-1)\). Let \(\sigma_t(h^t-1, G)\) assign probability 1 to \(g\) and let \(\sigma_t(h^t-1, B)\) assign probability 1 to \(b\).

Case 2: \(1 \notin \Phi_G(h^t-1)\) but \(0 \notin \Phi_B(h^t-1)\). By Bayesian plausibility, there exists some smallest \(q^* \in (0, \pi_{t-1})\) with \(q^* \in \Phi_G(h^t-1) \cap \Phi_B(h^t-1)\), induced by some message \(\bar{m}_b \in \bar{M}\) sent with positive probabilities in both states. Also, some message \(\bar{m}_g \in \bar{M}\) sent with positive probability in state \(G\) induces belief 1. Let \(\sigma_t(h^t-1, B)(b) = 1\) and let \(\sigma_t(\emptyset, G)(b) = x\) where \(x \in (0, 1)\) solves \(\frac{\pi_{t-1}^x}{\pi_{t-1}^x + (1-\pi_{t-1})} = q^*\).

Case 3: \(1 \notin \Phi_G(h^t-1)\) but \(0 \in \Phi_B(h^t-1)\). By Bayesian plausibility, there exists some largest \(q^* \in (\pi_{t-1}, 1)\) with \(q^* \in \Phi_G(h^t-1) \cap \Phi_B(h^t-1)\). Let \(\sigma_t(h^t-1, G)(g) = 1\) and let \(\sigma_t(h^t-1, B)(g) = x\) where \(x \in (0, 1)\) solves \(\frac{\pi_{t-1}^x}{\pi_{t-1}^x + (1-\pi_{t-1})} = q^*\).

Case 4: \(1 \notin \Phi_G(h^t-1)\) and \(0 \notin \Phi_B(h^t-1)\). By Bayesian plausibility, \(\Phi_G(h^t-1) = \Phi_B(h^t-1)\), and there exist some largest \(q_L \leq \pi_{t-1}\) and smallest \(q_H \geq \pi_{t-1}\) in this common set of posterior beliefs, and further there exist \(x, y \in (0, 1)\) so that \(\frac{\pi_{t-1}^x}{\pi_{t-1}^x + (1-\pi_{t-1})} = q_H\) and \(\frac{\pi_{t-1}^y}{\pi_{t-1}^y + (1-\pi_{t-1})} = q_L\). Let \(\sigma(h^t-1, G)(g) = x\) and \(\sigma(h^t-1, B)(g) = y\).

Having constructed \(\sigma_t\), let \(H_{\text{int}}^t\) be those on-path period \(t\) histories with interior equilibrium beliefs, that is \(h^t = (h^t-1, m) \in H_{\text{int}}^t\) if and only if \(h^t-1 \in H_{\text{int}}^t\) and \(\sigma(h^t-1, \theta)(m) > 0\) for both \(\theta \in \{G, B\}\). A property of the construction of \(\sigma_t\) is that if \(h^t \in H_{\text{int}}^t\), then both \((h^t-1, g)\) and \((h^t-1, b)\) are on-path. That is, off-path histories can only be continuations of
histories with degenerate beliefs in \( \{0, 1\} \).

Let \( \bar{H}_{int}^t \) be on-path period \( t \) histories with interior equilibrium beliefs in \((\bar{M}, \bar{\sigma})\). By the definition of \( \sigma_t \), there exists \( \bar{m} \in \bar{M} \) so that \( h^t \) induces the same equilibrium belief in the new equilibrium as the history \((\phi(h^{t-1}), \bar{m}) \in \bar{H}_{int}^t \) in the old equilibrium, and we define \( \phi(h^t) := (\phi(h^{t-1}), \bar{m}) \).

The receiver’s expected payoff in both the \( B \) and \( G \) states are the same as in the old equilibrium. To see this, note that by our construction, the receiver’s expected payoff in state \( B \) is the same as if we took a deterministic selection of messages \( m_1, m_2, \ldots \) in the old equilibrium with the property that \( \sigma_1(\emptyset, B)(m_1) > 0 \) and, for \( t \geq 2 \), \( \sigma_t(m_1, \ldots, m_{t-1}, \theta)(m_t) > 0 \). Then, we had the sender play message \( m_t \) in period \( t \). Since this sequence of messages is played with positive probability in state \( B \) of the old equilibrium, it must yield the expected payoff under \( B \) — if it yields higher or lower payoffs, then we can construct a deviation that improves the receiver’s ex-ante expected payoffs in the old equilibrium. A similar argument holds for state \( G \).

It remains to check that \((\bar{M}, \bar{\sigma})\) is an equilibrium by ruling out one-shot deviations. We argued before that all off-path histories must follow an on-path history with equilibrium belief in 0 or 1. There are no profitable deviations at off-path histories or at on-path histories with degenerate beliefs, because the receiver does not update beliefs after such histories regardless of the sender’s play.

So consider an on-path history with a non-degenerate belief, i.e. a member \( h^t \in \bar{H}_{int}^t \). A one-shot deviation following \( h^t \) corresponds to a deviation following \( \phi(h^t) \) in \((\bar{M}, \bar{\sigma})\), and must not be strictly profitable.

**Part 2.** We now turn to the second claim. If \( T \leq T' \), then for any equilibrium with horizon \( T \), we may construct an equilibrium of horizon \( T' \) which sends messages in the same way in periods 1, ..., \( T - 1 \), but babbles starting in period \( T \). This equilibrium has the same expected payoff as the old one.

\( \square \)

Note that the first claim of Lemma 1 also holds for the infinite horizon model of subsection 5.3. Nothing in the argument relies on \( T \) being finite. This is because the proof argument relies on the one-shot deviation property which holds for equilibria in both finite and infinite horizon models. Thus, in particular, in the proof of Proposition 15 we can also focus on a binary signal space.
A.13 Proof of Lemma 2

Proof. Due to sub-additivity,
\[ \mu(p) < \mu(p - \pi) + \mu(\pi). \] (2)
Note that symmetry implies \( \mu(-p) = -\mu(p) \) and that \( \mu(-\pi) = -\mu(\pi) \). Rearranged (2) is precisely \( N(0; \pi) < N(p; \pi) \).

□

A.14 Proof of Proposition 9

We begin by giving some additional definition and notation.

For \( p, \pi \in [0, 1] \), let \( N_G(p; \pi) := \mu(p - \pi) + \mu(1 - p) \).

We state and prove a preliminary lemma about \( N_G \) and \( N_B \).

Lemma A.2. Suppose \( \mu \) exhibits diminishing sensitivity and greater sensitivity to losses. Then, \( p \mapsto N_G(p; \pi) \) is strictly increasing on \([0, \pi]\) and symmetric on the interval \([\pi, 1]\). For each \( p_1 \in [\pi, 1] \), there exists exactly one point \( p_2 \in [\pi, 1] \) so that \( N_G(p_1; \pi) = N_G(p_2; \pi) \).

For every \( p_L < \pi \) and \( p_H \geq \pi \), \( N_G(p_L; \pi) < N_G(p_H; \pi) \). Also, \( N_B(p; \pi) \) is symmetric on the interval \([0, \pi]\). For each \( p_1 \in [0, \pi] \), there exists exactly one point \( p_2 \in [0, \pi] \) so that \( N_B(p_1; \pi) = N_B(p_2; \pi) \).

Proof. We have \( \frac{\partial N_G(p; \pi)}{\partial p} = \mu'(p - \pi) - \mu'(1 - p) \). For \( 0 \leq p < \pi \) and under greater sensitivity to losses, \( \mu'(p - \pi) \geq \mu'(\pi - p) \). Since \( \mu''(x) < 0 \) for \( x > 0 \), \( \mu'(\pi - p) > \mu'(1 - p) \). This shows \( \frac{\partial N_G(p; \pi)}{\partial p} > 0 \) for \( p \in [0, \pi) \).

The symmetry results follow from simple algebra and do not require any assumptions.

Note that \( \frac{\partial^2 N_G(p; \pi)}{\partial p^2} = \mu''(p - \pi) + \mu''(1 - p) < 0 \) for any \( p \in [\pi, 1] \), due to diminishing sensitivity. Combined with the required symmetry, this means \( \frac{\partial N_G(p; \pi)}{\partial p} \) crosses 0 at most once on \([\pi, 1]\), so for each \( p_1 \in [\pi, 1] \), we can find at most one \( p_2 \) so that \( N_G(p_1; \pi) = N_G(p_2; \pi) \). In particular, this implies at every intermediate \( p_1 \in (\pi, 1) \), we get \( N_G(p_1; \pi) > N_G(\pi; \pi) \) since we already have \( N_G(1; \pi) = N_G(\pi; \pi) \). This shows \( N_G(\cdot; \pi) \) is strictly larger on \([\pi, 1]\) than on \([0, \pi)\).

A similar argument, using \( \mu''(x) > 0 \) for \( x < 0 \), establishes that for each \( p_1 \in [0, \pi] \), we can find at most one \( p_2 \) so that \( N_B(p_1; \pi) = N_B(p_2; \pi) \).

□

Consider any period \( T - 2 \) history \( h_{T-2} \) in any equilibrium \((M, \sigma^*, p^*)\) where \( p^*(h_{T-2}) = \pi \in (0, 1) \). Let \( P_G \) and \( P_B \) represent the sets of posterior beliefs induced at the end of \( T - 1 \)
with positive probability, in the good and bad states. The next lemma gives an exhaustive enumeration of all possible $P_G, P_B$.

**Lemma A.3.** The sets $P_G, P_B$ belong to one of the following cases.

1. $P_G = P_B = \{\pi\}$
2. $P_G = \{1\}, P_B = \{0\}$
3. $P_G = \{p_1\}$ for some $p_1 \in (\pi, 1)$ and $P_B = \{0, p_1\}$
4. $P_G = \{\pi, 1\}$ and $P_B = \{0, \pi\}$
5. $P_G = \{p_1, p_2\}$ for some $p_1 \in (\pi, \frac{1+\pi}{2}), p_2 = 1 - p_1 + \pi, P_B = \{0, p_1, p_2\}$.

**Proof.** Suppose $|P_G| = 1$.

If $P_G = \{\pi\}$, then any equilibrium message not inducing $\pi$ must induce 0. By the Bayes’ rule, the sender cannot induce belief 0 with positive probability in the bad state, so $P_B = \{\pi\}$ as well.

If $P_G = \{1\}$, then any equilibrium message not inducing 1 must induce 0. Furthermore, the sender cannot send equilibrium messages inducing belief 1 with positive probability in the bad state, else the equilibrium belief associated with these messages should be strictly less than 1. Thus $P_B = \{0\}$.

If $P_G = \{p_1\}$ for some $0 \leq p_1 < \pi$, then any equilibrium message not inducing $p_1$ must induce 0. This is a contradiction since the posterior beliefs do not average out to $\pi$.

This leaves the case of $P_G = \{p_1\}$ for some $\pi < p_1 < 1$. Any equilibrium message not inducing $p_1$ must induce 0. Furthermore, the sender must induce the belief $p_1$ in the bad state with positive probability, else we would have $p_1 = 1$. At the same time, the sender must also induce belief 0 with positive probability in the bad state, else we violate Bayes’ rule. So $P_B = \{0, p_1\}$.

Now suppose $|P_G| = 2$.

In the good state, the sender must be indifferent between two beliefs $p_1, p_2$ both induced with positive probability. By Lemma A.2, $N_G(p; \pi)$ is strictly increasing on $[0, \pi]$ and strictly higher on $[\pi, 1]$ than on $[0, \pi)$, while for each $p_1 \in [\pi, 1]$, there exists exactly one point $p_2 \in [\pi, 1]$ so that $N_G(p_1; \pi) = N_G(p_2; \pi)$. This means we must have $p_1 \in [\pi, \frac{1+\pi}{2}], p_2 = 1 - p_1 + \pi$. 

52
If $P_G = \{\pi, 1\}$, any equilibrium message not inducing $\pi$ or 1 must induce 0. Also, $1 \not\in P_B$, because any message sent with positive probability in the bad state cannot induce belief 1. We cannot have $P_B = \{0\}$, because then the message inducing belief $\pi$ actually induces 1. We cannot have $P_B = \{\pi\}$ for then we violate Bayes’ rule. This leaves only $P_B = \{0, \pi\}$.

If $P_G = \{p_1, p_2\}$ for some $p_1 \in (\pi, \frac{1+\pi}{2})$, then any equilibrium message not inducing $p_1$ or $p_2$ must induce 0. Also, $p_1, p_2 \in P_B$, else messages inducing these beliefs give conclusive evidence of the good state. By Bayes’ rule, we must have $P_B = \{0, p_1, p_2\}$.

It is impossible that $|P_G| \geq 3$, since, by Lemma A.2, $N_G(p; \pi)$ is strictly increasing on $[0, \pi]$ and strictly higher on $[\pi, 1]$ than on $[0, \pi)$, while for each $p_1 \in [\pi, 1]$, there exists exactly one point $p_2 \in [\pi, 1]$ so that $N_G(p_1; \pi) = N_G(p_2; \pi)$. So the sender cannot be indifferent between 3 or more different posterior beliefs of the receiver in the good state.

We now give the proof of Proposition 9.

**Proof.** Consider any period $T-2$ history $h^{T-2}$ with $p^*(h^{T-2}) \in (0, 1)$. By Lemma 2, $N_B(p; p^*(h^{T-2})) > N_B(0; p^*(h^{T-2}))$ for all $p \in (p^*(h^{T-2}), 1]$. Therefore, cases 3 and 5 are ruled out from the conclusion of Lemma A.3. This shows that after having reached history $h^{T-2}$, the receiver will get total news utility of $\mu(1 - p^*(h^{T-2}))$ in the good state and $\mu(-p^*(h^{T-2}))$ in the bad state. This conclusion applies to all period $T-2$ histories (including those with equilibrium beliefs 0 or 1). So, the sender gets the same utility as if the state is perfectly revealed in period $T-1$ rather than $T$, and the equilibrium up to period $T-1$ form an equilibrium of the cheap talk game with horizon $T-1$. By backwards induction, we see that along the equilibrium path, whenever the receiver’s belief updates, it is updated to the dogmatic belief in $\theta$. \qed

### A.15 Proof of Proposition 10

**Proof.** The conclusions of Lemmas A.2 and A.3 continue to hold, since these only depend on $\mu$ exhibiting greater sensitivity to losses. As in the proof of Proposition 9, we only need to establish $N_B(p; \pi_0) > N_B(0; \pi_0)$ for all $p \in (\pi_0, 1]$ to rule out cases 3 and 5 from Lemma A.3 and hence establish our result.

For $p = \pi_0 + z$ where $z \in (0, 1 - \pi_0]$,

$$N_B(p; \pi_0) - N_B(0; \pi_0) = \mu(z) + \mu(-(\pi_0 + z)) - \mu(-\pi_0).$$
Consider the RHS as a function $D(z)$ of $z$. Clearly $D(0) = 0$, and $D'(z) = \mu'(z) - \mu'(-(\pi_0 + z))$. Since $\min_{z \in [0, 1 - \pi_0]} \frac{\mu'(z)}{\mu(-(\pi_0 + z))} > 1$, we get $D'(z) > 0$ for all $z \in [0, 1 - \pi_0]$, thus $D(z) > 0$ on the same range.

\section*{A.16 Proof of Corollary 4}

\textit{Proof.} First, $\mu$ exhibits greater sensitivity to losses, because $\mu(-x) = -\lambda \mu(x)$ for all $x > 0$ and we have $\lambda \geq 1$.

To apply Proposition 10, we only need to verify that $\min_{z \in [0, 1 - \pi_0]} \frac{\mu'(z)}{\mu(-(\pi_0 + z))} > 1$. For the $\lambda$-scaled $\mu$, $\min_{z \in [0, 1 - \pi_0]} \frac{\mu'(z)}{\mu(-(\pi_0 + z))} = \frac{1}{\lambda} \cdot \min_{z \in [0, 1 - \pi_0]} \frac{\tilde{\mu}'(z)}{\tilde{\mu}(\pi_0 + z)}$. The assumption that $\min_{z \in [0, 1 - \pi_0]} \frac{\tilde{\mu}'(z)}{\tilde{\mu}(\pi_0 + z)} > \lambda$ gives the desired conclusion.

\section*{A.17 Proof of Proposition 11}

\textit{Proof.} By the proof of Proposition 12, which does not depend on this result, there is a GGN equilibrium with one intermediate belief $p \in (\pi_0, 1)$ whenever $N_B(p; \pi_0) = N_B(0; \pi_0)$. In this equilibrium, the sender induces a belief of either $p$ or $0$ by the end of period 1, then babbles in all remaining periods of communication. Since the sender is indifferent between inducing belief $p$ or $0$ in the bad state, this equilibrium gives the same payoff as the babbling one in the bad state. But, since $\mu(p - \pi_0) + \mu(1 - p) > \mu(1 - \pi_0)$ due to strict concavity of $\tilde{\mu}_{pos}$, the receiver gets strictly higher news utility in the good state.

To find $\tilde{\lambda}$ that guarantees the existence of a $p$ solving $N_B(p; \pi_0) = N_B(0; \pi_0)$, let $D(p) := N_B(p; \pi_0) - N_B(0; \pi_0)$. We have $D(\pi_0) = 0$ and $\lim_{p \to \pi_0^+} D'(p) = \lim_{x \to 0^+} \tilde{\mu}'_{pos}(x) - \mu'(-\pi_0) = \lim_{x \to 0^+} \tilde{\mu}'_{pos}(x) - \lambda \mu' (\pi_0)$. For any finite $\lambda$, this limit is $\infty$, since $\lim_{x \to 0^+} \tilde{\mu}'_{pos}(x) = \infty$. On the other hand, $D(1) = \mu(1 - \pi_0) + \mu(-1) - \mu(-\pi_0) = \tilde{\mu}_{pos}(1 - \pi_0) - \lambda (\tilde{\mu}_{pos}(1) - \tilde{\mu}_{pos}(\pi_0))$. Since $\tilde{\mu}_{pos}(1) - \tilde{\mu}_{pos}(\pi_0) > 0$, we may find a large enough $\tilde{\lambda} \geq 1$ so that $\tilde{\mu}_{pos}(1 - \pi_0) - \tilde{\lambda} (\tilde{\mu}_{pos}(1) - \tilde{\mu}_{pos}(\pi_0)) < 0$. Whenever $\lambda \geq \tilde{\lambda}$, we therefore get $D(\pi_0) = 0$, $\lim_{p \to \pi_0^+} D'(p) = \infty$, and $D(1) < 0$. By the intermediate value theorem applied to the continuous $D$, there exists some $p \in (\pi_0, 1)$ so that $D(p) = 0$.\qed
A.18 Proof of Proposition 12

Proof. Let $J$ intermediate beliefs satisfying the hypotheses be given. We construct a gradual good news equilibrium where $p_t = q^{(i)}$ for $1 \leq t \leq J$, and $p_t = q^{(J)}$ for $J + 1 \leq t \leq T - 1$.

Let $M = \{g, b\}$ and consider the following strategy profile. In period $t \leq J$ where the public history so far $h^{t-1}$ does not contain any $b$, let $\sigma(h^{t-1}; G)(g) = 1$, $\sigma(h^{t-1}; B)(g) = x$ where $x \in (0,1)$ satisfies $\frac{p_{t-1}}{p_{t-1} + (1-p_{t-1})} = p_t$. But if public history contains at least one $b$, then $\sigma(h^{t-1}; G)(b) = 1$ and $\sigma(h^{t-1}; B)(b) = 1$. Finally, if the period is $t > J$, then $\sigma(h^{t-1}; G)(b) = 1$ and $\sigma(h^{t-1}; B)(b) = 1$. In terms of beliefs, suppose $h^{t}$ has $t \leq J$ and every message so far has been $g$. Such histories are on-path and get assigned the Bayesian posterior belief. If $h^{t}$ has $t \leq J$ and contains at least one $b$, then it gets assigned belief 0. Finally, if $h^{t}$ has $t > J$, then $h^{t}$ gets assigned the same belief as the subhistory constructed from its first $J$ elements. It is easy to verify that these beliefs are derived from Bayes’ rule whenever possible.

We verify that the sender has no incentive to deviate. Consider period $t \leq J$ with history $h^{t-1}$ that does not contain any $b$. The receiver’s current belief is $p_{t-1}$ by construction.

In state $B$, we first calculate the sender’s equilibrium payoff after sending $g$. The receiver will get some $I$ periods of good news before the bad state is revealed, either by the sender or by nature in period $T$. That is, the equilibrium news utility with $I$ periods of good news is given by

$$\sum_{i=1}^{I} \mu(p_{t-1+i} - p_{t-2+i}) + \mu(-p_{t-1+i}).$$

Since $p_{t-1+i} \in P^*(p_{t-2+i})$, we have $N_B(p_{t-1+i}; p_{t-2+i}) = N_B(0; p_{t-2+i})$, that is to say $\mu(p_{t-1+i} - p_{t-2+i}) + \mu(-p_{t-1+i}) = \mu(-p_{t-2+i})$. We may therefore rewrite the receiver’s total news utility as $\sum_{i=1}^{I-1} \mu(p_{t-1+i} - p_{t-2+i}) + \mu(-p_{t-2+i})$. But by repeating this argument, we conclude that the receiver’s total news utility is just $\mu(-p_{t-1})$. Since this result holds regardless of $I$’s realization, the sender’s expected total utility from sending $g$ today is $\mu(-p_{t-1})$, which is the same as the news utility from sending $b$ today. Thus, sender is indifferent between $g$ and $b$ and has no profitable deviation.

In state $G$, the sender gets at least $\mu(1 - p_{t-1})$ from following the equilibrium strategy. This is because the receiver’s total news utility in the good state along the equilibrium path is given by $\sum_{i=1}^{I-1} \mu(p_{t-1+i} - p_{t-2+i}) + \mu(1 - p_{t-1+i})$. By sub-additivity in gains, this sum is strictly larger than $\mu(1 - p_{t-1})$. If the sender deviates to sending $b$ today, then the receiver
updates belief to 0 today and belief remains there until the exogenous revelation, when belief updates to 1. So this deviation gives the total news utility $\mu(-p_{t-1}) + \mu(1)$. We have

$$
\mu(1) < \mu(1 - p_{t-1}) + \mu(p_{t-1}) \\
\leq \mu(1 - p_{t-1}) - \mu(-p_{t-1}),
$$

where the first inequality comes from sub-additivity in gains, and the second from weak loss aversion. This shows $\mu(-p_{t-1}) + \mu(1) < \mu(1 - p_{t-1})$, so the deviation is strictly worse than sending the equilibrium message.

Finally, at a history containing at least one $b$ or a history with length $K$ or longer, the receiver’s belief is the same at all continuation histories. So the sender has no deviation incentives since no deviations affect future beliefs.

For the other direction, suppose by way of contradiction there exists a gradual good news equilibrium with the $J$ intermediate beliefs $q^{(1)} < ... < q^{(J)}$. For a given $1 \leq j \leq J$, find the smallest $t$ such that $p_t = q^{(k-1)}$ and $p_{t+1} = q^{(k)}$. At every on-path history $h^t \in H^t$ with $p^*(h^t) = p_t$, we must have $\sigma^*(h^t; B)$ inducing both 0 and $q^{(j)}$ with strictly positive probability. Since we are in equilibrium, we must have $\mu(-q^{(j-1)})$ being equal to $\mu(q^{(j)} - q^{(j-1)})$ plus the continuation payoff. If $j = J$, then this continuation payoff is $\mu(-q^{(j)})$ as the only other period of belief movement is in period $T$ when the receiver learns the state is bad. If $j < J$, then find the smallest $\bar{t}$ so that $p_{\bar{t}+1} = q^{(j+1)}$. At any on-path $h^\bar{t} \in H^\bar{t}$ which is a continuation of $h^t$, we have $p^*(h^\bar{t}) = q^{(j)}$ and the receiver has not experienced any news utility in periods $t + 2, ..., \bar{t}$. Also, $\sigma^*(h^\bar{t}; B)$ assigns positive probability to inducing posterior belief 0, so the continuation payoff in question must be $\mu(-q^{(j)})$. So we have shown that $\mu(-q^{(j-1)}) = \mu(q^{(j)} - q^{(j-1)}) + \mu(-q^{(j)})$, that is $N_B(q^{(j)}; q^{(j-1)}) = N_B(0; q^{(j-1)})$.

A.19 Proof of Corollary 5

Proof. We apply Proposition 12 to the case of quadratic. Recall the relevant indifference equation in the good state.

$$
(!) \quad \mu(-q_t) = \mu(q_{t+1} - q_t) + \mu(-q_{t+1}).
$$

Plugging in the quadratic specification and algebraic transformations lead to
\[ 0 = (\alpha_p - \alpha_n)(q_{t+1} - q_t) - \beta_p(q_{t+1} - q_t) + \beta_n(q_{t+1} - q_t)(q_{t+1} + q_t) \]

Define \( r = q_{t+1} - q_t \). Then this relation can be written as

\[(\beta_p - \beta_n)r^2 + (\alpha_n - \alpha_p - 2\beta_nq_t)r = 0,\]

i.e. \( r \) is a zero of a second order polynomial. For \( P^* \) to be non-empty we need this root \( r \) to be in \((0, 1 - q_t)\). In particular the peak/trough \( \bar{r} \) of the parabola defined by the second order polynomial should satisfy \( \bar{r} \in (0, \frac{1-q_t}{2}) \). Given that \( \bar{r} = \frac{2\beta_nq_t - (\alpha_n - \alpha_p)}{2(\beta_p - \beta_n)} \) for the case that \( \beta_p \neq \beta_n \), we get the equivalent condition on the primitives

\[ 0 < \frac{2\beta_nq_t - (\alpha_n - \alpha_p)}{2(\beta_p - \beta_n)} < \frac{1-q_t}{2}. \]

The root \( r \) itself is given by \( r = \frac{2\beta_nq_t - (\alpha_n - \alpha_p)}{\beta_p - \beta_n} \), which leads to the recursion

\[(R) \quad q_{t+1} = q_t \frac{\beta_p + \beta_n}{\beta_p - \beta_n} - \frac{\alpha_n - \alpha_p}{\beta_p - \beta_n}. \]

This leads to the formula for \( P^*(\pi) \) in part 1).

**Case 1:** When \( \beta_p < \beta_n \) the coefficient in front of \( q_t \) is negative so that the recursion \((R)\) leads to

\[ (!!) \quad q_{t+1} - q_t = q_t \frac{2\beta_n}{\beta_p - \beta_n} - \frac{\alpha_n - \alpha_p}{\beta_p - \beta_n} < 0. \]

One also sees here that for the case that \( \beta_p < \beta_n \) to give a gradual good news equilibrium of time-length 1, one needs a low enough prior: namely \( \pi_0 < \frac{\alpha_n - \alpha_p}{2\beta_n} =: q^* \). For all priors larger or equal than \( q^* \), there is no one-shot bad news partial good news equilibrium.

**Case 2:** When \( \beta_p > \beta_n \) the slope in \((R)\) is above 1 so that for all priors \( \pi_0 \) large enough we get an increasing sequence \( q_t \) which satisfies \((!\)\). It is also easy to see from \((R)\) that

\[ (q_{t+2} - q_{t+1}) - (q_{t+1} - q_t) = \left( \frac{\beta_p + \beta_n}{\beta_p - \beta_n} - 1 \right) > 0, \]

proving the statement in the text after the corollary.

That an equilibrium can exist where partial good news are released for more than two periods, is shown by the example in the main text following the statement of the Corollary (see Figure 5).
A.20 Proof of Proposition 13

Proof. Since $N_B(p; \pi) - N_B(0; \pi) = 0$ for $p = \pi$ and $\frac{\partial}{\partial p} N_B(p; \pi)|_{p=\pi} > 0$, $N_B(p; \pi) - N_B(0; \pi)$ starts off positive for $p$ slightly above $\pi$. Given that $|P^*(\pi)| \leq 1$, if we find some $p' > \pi$ with $N_B(p'; \pi) - N_B(0; \pi) > 0$, then any solution to $N_B(p; \pi) - N_B(0; \pi) = 0$ in $(\pi, 0)$ must lie to the right of $p'$.

If $q^{(j)}, q^{(j+1)}$ are intermediate beliefs in a GGN equilibrium, then by Proposition 12, $q^{(j)} \in P^*(q^{(j-1)})$ and $q^{(j+1)} \in P^*(q^{(j)})$. Let $p' = q^{(j)} + (q^{(j)} - q^{(j-1)})$. Then,

$$N_B(p'; q^{(j)}) - N_B(0; q^{(j)}) = \mu(p' - q^{(j)}) + \mu(-p') - \mu(-q^{(j)})$$

$$= \mu(q^{(j)} - q^{(j-1)}) + \mu(-q^{(j)} - (q^{(j)} - q^{(j-1)})) - \mu(-q^{(j)})$$

$$> \mu(q^{(j)} - q^{(j-1)}) + \mu(-q^{(j-1)} - (q^{(j)} - q^{(j-1)})) - \mu(-q^{(j-1)})$$

where the last inequality comes from diminishing sensitivity. But, the final expression is $N_B(q^{(j)}; q^{(j-1)}) - N_B(0; q^{(j-1)})$, which is 0 since $q^{(j)} \in P^*(q^{(j-1)})$. This shows we must have $q^{(j+1)} - q^{(j)} > q^{(j)} - q^{(j-1)}$. \qed

A.21 Proof of Corollary 6

Proof. We verify the sufficient condition in Proposition 13. We get $\frac{\partial}{\partial p} N_B(p; \pi) = \frac{\alpha}{(p-\pi)^{1-\alpha}} - \frac{\lambda}{p-\pi}$, so $\frac{\partial}{\partial p} N_B(p; \pi)|_{p=\pi} = \infty$.

To show that $|P^*(\pi)| \leq 1$, it suffices to show that $\frac{\partial}{\partial p} N_B(p; \pi) = 0$ for at most one $p > \pi$. For the derivative to be zero, we need $(\frac{\pi}{p-\pi})^{1-\alpha} = \lambda$. As the LHS is decreasing for $p > \pi$, it can have at most one solution. \qed

A.22 Proof of Proposition 14

Proof. Consider the following operator $\phi$ on the space of continuous functions on $[0, 1]$. For $V : [0, 1] \to \mathbb{R}$, define $\phi(V)(p) := \tilde{V}(p \mid p)$, where

$$\tilde{V}(\cdot \mid p) := \text{cav}_q[\mu(q - p) + \delta V(q) + (1 - \delta)(q \cdot \mu(1 - q) + (1 - q) \cdot \mu(-q))].$$

We show that $\phi$ satisfies the Blackwell conditions and so is a contraction mapping.
Suppose that $V_2 \geq V_1$ pointwise. Then for any $p, q \in [0, 1]$,

$$
\mu(q-p)+\delta V_2(q)+(1-\delta)(q\mu(1-q)+(1-q)\mu(-q)) \geq (q-p)+\delta V_1(q)+((1-\delta)(q\mu(1-q)+(1-q)\mu(-q))
$$

therefore $\tilde{V}_2(\cdot \mid p) \geq \tilde{V}_1(\cdot \mid p)$ pointwise as well. In particular, $\tilde{V}_2(p \mid p) \geq \tilde{V}_1(p \mid p)$, that is $\phi(V_2)(p) \geq \phi(V_1)(p)$.

Also, let $k > 0$ be given and let $V_2 = V_1 + k$ pointwise. It is easy to see that $\tilde{V}_2(\cdot \mid p) = \tilde{V}_1(\cdot \mid p) + \delta k$ for every $p$, because the argument to the concavification operator will be pointwise higher by $\delta k$. So in particular, $\phi(V_2)(p) = \phi(V_1)(p) + \delta k$. By the Blackwell conditions, the operator $\phi$ is a contraction mapping on the metric space of continuous functions on $[0, 1]$ with the supremum norm. Thus, the value function exists and is also unique.

To show pointwise monotonicity in $\delta$, suppose $0 \leq \delta < \delta' < 1$. First, $V_\delta(0) = V_\delta(1) = 0$ for any $\delta \in [0, 1)$. Now consider an environment where full revelation happens at the end of each period with probability $1-\delta$, and fix a prior $p \in (0, 1)$. There exists some binary information structure with message space $M = \{0, 1\}$, public histories $H^t = (M)^t$ for $t = 0, 1, \ldots$, and sender strategies $(\sigma_t)_{t=0}^\infty$ with $\sigma_t : H^t \times \Theta \rightarrow \Delta(M)$, such that $(M, \sigma)$ induces expected news utility of $V_\delta(p)$ when starting at prior $p$.

We now construct a new information structure, $(\tilde{M}, \tilde{\sigma})$ to achieve expected news utility $V_\delta(p)$ when starting at prior $p$ in an environment where full revelation happens at the end of each period with probability $1-\delta'$, with $\delta' > \delta$. Let $\tilde{M} = \{0, 1, \emptyset\}$. The idea is that when full revelation has not happened, there is a $1 - \frac{\delta}{\delta'}$ probability each period that the sender enters into a babbling regime forever. When the sender enters the babbling regime at the start of period $t + 1$, the receiver’s expected utility going forward is the same as if full revelation happened at the start of $t + 1$.

To implement this idea, after any history $h^t \in H^t$ not containing $\emptyset$, let

$$
\tilde{\sigma}_{t+1}(h^t; \theta) = \begin{cases} 
\emptyset & \text{w/p } 1 - \frac{\delta}{\delta'} \\
1 & \text{w/p } \frac{\delta}{\delta'} \cdot \sigma_{t+1}(h^t; \theta)(1) \\
0 & \text{w/p } \frac{\delta}{\delta'} \cdot \sigma_{t+1}(h^t; \theta)(0)
\end{cases}
$$

That is, conditional on not entering the babbling regime, $\tilde{\sigma}$ behaves in the same way as $\sigma$. But, after any history $h^t \in H^t$ containing at least one $\emptyset$, $\tilde{\sigma}_{t+1}(h^t; \theta) = \emptyset$ with probability

59
1. Once the sender enters the babbling regime, she babbles forever (until full revelation exogenously arrives at some random date). We need to verify that payoff from this strategy is indeed \( V_h(p) \). Fix a history \( h^t \) not containing \( \emptyset \) and a state \( \theta \), and suppose \( p^*(h^t) = q \). Under \( \bar{\sigma}_{t+1} \), with probability of \( (1 - \delta') + \delta'(1 - \frac{\delta}{\bar{\delta}}) = 1 - \delta \) the receiver gets the expected babbling payoff \( q\mu(1 - q) + (1 - q)\mu(-q) \) in the period of state revelation. Analogously, under \( \sigma_{t+1} \), there is probability \( 1 - \delta \) that state revelation happens in period \( t + 1 \) and the receiver gets \( q\mu(1 - q) + (1 - q)\mu(-q) \) in expectation. With probability \( \delta', \frac{\delta}{\bar{\delta}} = \delta \), the receiver facing \( \bar{\sigma} \) gets the payoff induced by \( \sigma_{t+1}(h^t; \theta) \) in period \( t + 1 \) and the same distribution of continuation histories as under \( \sigma \). The same argument applies to all these continuation histories, so \( \bar{\sigma} \) must induce the same expected payoff as \( \sigma \) when starting at \( (h^t; \theta) \).

\[ \Box \]

A.23 Proof of Proposition 15

Proof. We show first sufficiency. Consider the following strategy profile. In period \( t \) where the public history so far \( h^{t-1} \) does not contain any \( b \), let \( \sigma(h^{t-1}; G)(g) = 1 \), \( \sigma(h^{t-1}; B)(g) = x \) where \( x \in (0, 1) \) satisfies \( \frac{p_{t-1} \cdot x}{p_{t-1} + (1 - p_{t-1})x} = p_t \). But if public history contains at least one \( b \), then \( \sigma(h^{t-1}; G)(b) = 1 \) and \( \sigma(h^{t-1}; B)(b) = 1 \). In terms of beliefs, suppose \( h^t \) is so that every message so far has been \( g \). Such histories are on-path and get assigned the Bayesian posterior belief. If \( h^t \) contains at least one \( b \), then belief is 0. It is easy to verify that these beliefs are derived from Bayes’ rule whenever possible.

We verify that the sender has no incentive to deviate. Consider period \( t \) with history \( h^{t-1} \) that does not contain any \( b \). The receiver’s current belief is \( p_{t-1} \) by construction.

In state \( B \), we first calculate the sender’s equilibrium payoff after sending \( g \). For any realization of the exogenous revelation date, the receiver’s total news utility in the good state along the equilibrium path is given by \( \sum_{j=1}^J \mu(p_{t-1+j} - p_{t-2+j}) + \mu(-p_{t-1+j}) \) for some integer \( J \geq 1 \). Since \( p_{t-1+j} \in \mathcal{P}^*(p_{t-2+j}) \), we have \( N_B(p_{t-1+j}; p_{t-2+j}) = N_B(0; p_{t-2+j}) \), that is to say \( \mu(p_{t-1+j} - p_{t-2+j}) + \mu(-p_{t-1+j}) = \mu(-p_{t-2+j}) \). We may therefore rewrite the receiver’s total news utility as \( \sum_{j=1}^{J-1} \mu(p_{t-1+j} - p_{t-2+j}) + \mu(-p_{t-2+j}) \). But by repeating this argument, we conclude that the receiver’s total news utility is just \( \mu(-p_{t-1}) \). Since this result holds regardless of \( J \), the sender’s expected total utility from sending \( g \) today is \( \mu(-p_{t-1}) \), which is the same as the news utility from sending \( b \) today. Thus, sender is indifferent between \( g \) and \( b \) and has no profitable deviation.

In state \( G \), the sender gets at least \( \mu(1 - p_{t-1}) \) from following the equilibrium strategy.
This is because for any realization of the exogenous revelation date, the receiver’s total news utility in the good state along the equilibrium path is given by \( \sum_{j=1}^{J} \mu(p_{t-1+j} - p_{t-2+j}) + \mu(1 - p_{t-1+j}) \) for some integer \( J \geq 1 \). By sub-additivity in gains, this sum is strictly larger than \( \mu(1 - p_{t-1}) \). If the sender deviates to sending \( b \) today, then the receiver updates belief to 0 today and belief remains there until the exogenous revelation, when belief updates to 1. So this deviation has the total news utility \( \mu(-p_{t-1}) + \mu(1) \). We have

\[
\mu(1) < \mu(1 - p_{t-1}) + \mu(p_{t-1}) \leq \mu(1 - p_{t-1}) - \mu(-p_{t-1}),
\]

where the first inequality comes from sub-additivity in gains, and the second from weak loss aversion. This shows \( \mu(-p_{t-1}) + \mu(1) < \mu(1 - p_{t-1}) \), so the deviation is strictly worse than sending the equilibrium message.

Finally, at a history containing at least one \( b \), the receiver’s belief is the same at all continuation histories. So the sender has no deviation incentives since no deviations affect future beliefs.

We now show necessity. Suppose that we have a (possibly infinite) gradual good news equilibrium given by the sequence \( p_0 < p_1 < \cdots < p_t < \ldots \). By Bayesian plausibility and because we are focusing on two-message equilibria the sender must be sending the messages \( \{0, p_t\} \) in period \( t \) if the state is bad. The sender must thus be indifferent between these two posteriors in the bad state. Formally, \( N_B(0; p_t) = N_B(p_{t+1}; p_t) \) for all \( t \geq 0 \), as long as there is no babbling. Written equivalently in the language of \( P^* \): \( p_{t+1} \in P^*(p_t) \) for all \( t \geq 0 \), as long as there’s no babbling, where here \( p_0 = \pi_0 \).

\[ \Box \]

**B Residual Consumption Uncertainty**

**B.1 A Model of Residual Consumption Uncertainty**

In the main text, we studied a model where the sender has perfect information about the receiver’s final-period consumption level.

Now suppose the sender’s information is imperfect. In state \( \theta \), the receiver will consume a random amount \( c \) in period \( T + 1 \), drawn as \( c \sim F_\theta \), deriving from it consumption utility \( v(c) \). As before, \( v \) is a strictly increasing consumption-utility function. We interpret the
state $\theta$ as the sender’s private information about the receiver’s future consumption, while the distribution $F_\theta$ captures the receiver’s residual consumption uncertainty conditional on what the sender knows. The case where $F_\theta$ is degenerate for every $\theta \in \Theta$ nests the baseline model.

Assume that $\mathbb{E}_{c \in F_\theta}[v(c)] \neq \mathbb{E}_{c \in F_\theta'}[v(c)]$ when $\theta' \neq \theta''$. We may without loss normalize $\min_{\theta \in \Theta} \mathbb{E}_{c \in F_\theta}[v(c)] = 0$, $\max_{\theta \in \Theta} \mathbb{E}_{c \in F_\theta}[v(c)] = 1$.

The mean-based news-utility function $N(\pi_t | \pi_{t-1})$ in this environment is the same as in the environment where the receiver always gets consumption utility $\mathbb{E}_{c \sim F_\theta}[v(c)]$ in state $\theta$. This is because given a pair of beliefs $F_{\text{old}}, F_{\text{new}} \in \Delta(\Theta)$ about the state, the receiver derives news utility $N(F_{\text{new}} | F_{\text{old}})$ based on the difference in expected consumption utilities, $\mu(\mathbb{E}_{c \sim F_{\text{new}}}[v(c)] - \mathbb{E}_{c \sim F_{\text{old}}}[v(c)])$. So, all of the results in the paper concerning mean-based news utility immediately extend. The two results in the paper that are not specific to mean-based news utility, Propositions 1 and 2, apply to any functions $N(\pi_t | \pi_{t-1})$ satisfying the continuous differentiability condition stated in Section 2, without requiring any relationship between $N$ and consumptions in different states.

We now define $N$ using Kőszegi and Rabin (2009)’s percentile-based news-utility model with a power-function gain-loss utility, in an environment with residual consumption uncertainty. We apply Proposition 2 to the resulting $N$ and show that one-shot resolution is strictly sub-optimal. This result applies for any $K \geq 2$.

**Corollary A.1.** Consider the percentile-based model with $\mu(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\alpha & x < 0 \end{cases}$ for $0 < \alpha < 1$, $\lambda \geq 1$. Suppose there are two states $\theta_G, \theta_B \in \Theta$ with distributions of consumption utilities $v(F_{\theta_B}) = \text{Unif}[0, L]$, $v(F_{\theta_G}) = J + v(F_{\theta_B})$ for some $L, J > 0$. One-shot resolution is strictly suboptimal for any finite $T$.

**Proof.** We show that $\lim_{\epsilon \to 0} \frac{N(1_G|1_G \oplus \epsilon 1_B)}{\epsilon} = \infty$ under this set of conditions. The argument behind Proposition 2 then implies some information structure involving perfect revelation of states other than $\theta_G, \theta_B$, one-shot bad news, partial good news for the two states $\theta_G, \theta_B$ is strictly better than one-shot resolution.

For $r \in [0, 1]$, write $F_r$ for the distribution of consumption utilities under the belief $r 1_G \oplus (1-r) 1_B$.

Note we must have $\int_0^1 c_{F_1}(q) - c_{F_{1-\epsilon}}(q) dq = J \epsilon$, and that $c_{F_1}(q) - c_{F_{1-\epsilon}}(q) \geq 0$ for all $q$. 

62
Let $q^* = \min(\epsilon \cdot J/L, \epsilon)$. It is the quantile at which $c_{F_{1-\epsilon}}(q^*) = J$.

For all $q \geq q^*$, $c_{F_{1-\epsilon}}(q) - c_{F_{1-\epsilon}}(q) \leq \epsilon L$.

**Case 1:** $J \geq L$, so $q^* = \epsilon$.

$$\int_0^{q^*} c_{F_{1-\epsilon}}(q) - c_{F_{1-\epsilon}}(q) dq = \int_0^{\epsilon} J - q \cdot \frac{1}{\epsilon} \cdot (1 - \epsilon) L dq$$

$$= J\epsilon - \frac{1}{2} \epsilon (1 - \epsilon) L.$$

This implies $\int_0^{q^*} c_{F_{1-\epsilon}}(q) - c_{F_{1-\epsilon}}(q) dq = \frac{1}{2} \epsilon (1 - \epsilon) L$.

The worst case is when the difference is $\epsilon L$ on some $q$-interval, and 0 elsewhere. For small $\epsilon < 0$ so that $\epsilon L < 1$,

$$\int_{q^*}^{1} (c_{F_{1-\epsilon}}(q) - c_{F_{1-\epsilon}}(q))^\alpha dq \geq (\epsilon L)^\alpha \cdot \frac{(1/2) \cdot \epsilon (1 - \epsilon) L}{\epsilon L}$$

$$= \frac{1}{2} (\epsilon L)^\alpha (1 - \epsilon).$$

Therefore, for small $\epsilon > 0$, $\frac{N(1_G | (1-\epsilon) 1_G \oplus 1_B)}{\epsilon} = \frac{1}{2} \epsilon L^\alpha (1 - \epsilon)$, which diverges to $\infty$ as $\epsilon \to 0$.

**Case 2:** $J < L$, so $q^* = \epsilon J/L$.

$$\int_0^{\epsilon J/L} c_{F_{1-\epsilon}}(q) - c_{F_{1-\epsilon}}(q) dq = \int_0^{\epsilon J/L} J - q \cdot \frac{1}{\epsilon J/L} (J - \frac{J}{L} \epsilon \cdot L) dq$$

$$= \frac{1}{2} J^2 \epsilon + \frac{1}{2} J^2 \epsilon^2 L$$

$$< \frac{1}{2} J\epsilon + \frac{1}{2} L \epsilon^2$$

using $J < L$. This then implies $\int_0^{q^*} c_{F_{1-\epsilon}}(q) - c_{F_{1-\epsilon}}(q) dq > \frac{1}{2} J\epsilon - \frac{1}{2} L \epsilon^2$.

So, again using the worst-case of the difference being $\epsilon L$ on some $q$-interval, and 0 elsewhere,

$$\frac{N(1_G | (1-\epsilon) 1_G \oplus 1_B)}{\epsilon} \geq \frac{1}{\epsilon} (\epsilon L)^\alpha \cdot \frac{1}{2} J\epsilon - \frac{1}{2} L \epsilon^2$$

$$= \frac{1}{\epsilon^{1-\alpha}} L^\alpha \cdot \left( \frac{1}{2} J/L - \frac{1}{2} \epsilon \right).$$

As $\epsilon \to 0$, RHS converges to $\infty$. □
B.2 A Calibration Comparing Percentile-Based News Utility and Mean-Based News Utility

Since Proposition 1’s procedure for computing the optimal information structure applies to general $N$, including both the percentile-based and the mean-based news-utility functions in an environment with residual consumption uncertainty, we can compare the solutions to the sender’s problem for these two models.

Consider two states of the world, $\Theta = \{G, B\}$. For some $\sigma > 0$, suppose consumption is distributed normally conditional on $\theta$ with $F_G = \mathcal{N}(1, \sigma^2)$, $F_B = \mathcal{N}(0, \sigma^2)$, consumption utility is $v(x) = x$, and gain-loss utility (over consumption) is $\mu(x) = \sqrt{x}$ for $x \geq 0$, $\mu(x) = -1.5\sqrt{-x}$ for $x < 0$. We calculated the optimal information structure for the mean-based model in an analogous environment, as reported in Figure 2.

With the percentile-based model, an agent who believes $\mathbb{P}[\theta = G] = \pi$ has a belief over final consumption given by a mixture normal distribution, $\pi F_G \oplus (1 - \pi) F_B$, illustrated in Figure A.2.

We plot in Figure A.3 the optimal information structures for $T = 5$, $\sigma = 1$. The optimal information structures for $\sigma = 0.1, 1, 10$ all involve gradual good news, one-shot bad news. Table A.1 lists the optimal disclosure of good news over time. Not only are the shapes of the concavification problems qualitatively similar to those of the mean-based model, but the resulting optimal information structures also bear striking quantitative similarities.

<table>
<thead>
<tr>
<th></th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
<th>$t = 4$</th>
<th>$t = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>percentile-based, $\sigma = 0.1$</td>
<td>0.50</td>
<td>0.55</td>
<td>0.61</td>
<td>0.69</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>percentile-based, $\sigma = 1$</td>
<td>0.50</td>
<td>0.55</td>
<td>0.62</td>
<td>0.71</td>
<td>0.83</td>
<td>1.00</td>
</tr>
<tr>
<td>percentile-based, $\sigma = 10$</td>
<td>0.50</td>
<td>0.56</td>
<td>0.63</td>
<td>0.72</td>
<td>0.84</td>
<td>1.00</td>
</tr>
<tr>
<td>mean-based, any $\sigma$</td>
<td>0.500</td>
<td>0.556</td>
<td>0.626</td>
<td>0.715</td>
<td>0.834</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table A.1: Optimal disclosure of good news. The optimal information structure under a square-root gain-loss function with $\lambda = 1.5$ takes the form of gradual good news, one-shot bad news both in the mean-based model and the percentile-based model for $T = 5$, $\sigma = 0.1, 1, 10$. The table shows belief movements conditional on the good state in different periods.

From Table A.1, it appears that percentile-based and mean-based models deliver more similar results for larger $\sigma^2$. We provide an analytic result consistent with the idea that these two models generate similar amounts of news utility when the state-dependent consumption
Figure A.2: The densities and CDFs of final consumption utility distributions under two beliefs about $\mathbb{P}[\theta = G]$, $\pi = 0.1$ and $\pi = 0.9$. The dashed black lines in the CDFs plot show the differences in consumption utilities at the 25th percentile, 50th percentile, and 75th percentile levels between these two beliefs. The news utility associated with updating belief from $\pi = 0.1$ to $\pi = 0.9$ in the percentile-based model is calculated by applying a gain-loss function $\mu$ to all these differences in consumption utilities at various quantiles, then integrating over all quantiles levels in $[0, 1]$. 
Figure A.3: The concavifications giving the optimal information structure with horizon $T = 5$, gain-loss function $\mu(x) = \begin{cases} \sqrt{x} & \text{for } x \geq 0 \\ -1.5\sqrt{-x} & \text{for } x < 0 \end{cases}$, prior $\pi_0 = 0.5$, using Kőszegi and Rabin (2009)'s percentile-based model in a Gaussian environment with $\sigma = 1$. The $y$-axis in each graph shows the sum of news utility this period and the value function of entering next period with a certain belief.
utility distributions have large variances.

**Proposition 16.** Suppose $\Theta = \{B, G\}$ and the distributions of consumption utilities in states $B$ and $G$ are $\text{Unif}[0, L]$ and $\text{Unif}[d, L + d]$ respectively, for $L, d > 0$. Let $N^{perc}(p_2 \mid p_1)$ be the news utility associated with changing belief in $\theta = G$ from $p_1$ to $p_2$ in a percentile-based news-utility model with a continuous gain-loss utility $\mu$. Then,

$$\lim_{L \to \infty} \left( \sup_{0 \leq p_1, p_2 \leq 1} |N^{perc}(p_2 \mid p_1) - \mu[(p_2 - p_1)d]| \right) = 0.$$ 

In a uniform environment, if there is enough unresolved consumption risk even conditional on the state $\theta$, then the difference between percentile-based news utility and mean-based news utility goes to zero uniformly across all possible belief changes.\(^6\)

*Proof.* Let $F_p(x)$ be the distribution function of the mixed distribution $p \cdot \text{Unif}[d, L + d] \oplus (1 - p) \cdot \text{Unif}[0, L]$, and $F_p^{-1}(q)$ its quantile function for $q \in [0, 1]$. By a simple calculation, $F_p^{-1}(d/L) = d + pd$ and $F_p^{-1}(1 - d/L) = L + pd - d$. At the same time, for $d/L \leq q \leq 1 - d/L$ where $q = d/L + y$, we have $F_p^{-1}(q) = d + pd + yL$.

This shows that over the intermediate quantile values between $d/L$ and $1 - d/L$,

$$\int_{d/L}^{1-d/L} \mu \left[ F_{p_2}^{-1}(q) - F_{p_1}^{-1}(q) \right] dq = \int_{d/L}^{1-d/L} \mu \left[ (p_2 - p_1)d \right] dq = (1 - 2d/L) \cdot \mu[(p_2 - p_1)d].$$

For the lower part of the quantile integral $[0, d/L]$, using the fact that $F_p^{-1}(d/L) = d + pd$, we have the uniform bound $0 \leq F_p^{-1}(q) \leq 2d$ for all $p \in [0, 1]$ and $q \leq d/L$. So,

$$\left| \int_0^{d/L} \mu \left[ F_{p_2}^{-1}(q) - F_{p_1}^{-1}(q) \right] dq \right| \leq \frac{d}{L} \cdot \max_{x \in [-2d, 2d]} |\mu(x)|.$$ 

By an analogous argument,

$$\left| \int_{1-d/L}^1 \mu \left[ F_{p_2}^{-1}(q) - F_{p_1}^{-1}(q) \right] dq \right| \leq \frac{d}{L} \cdot \max_{x \in [-2d, 2d]} |\mu(x)|.$$ 

So for any $0 \leq p_1, p_2 \leq 1$,

$$|N^{perc}(p_2 \mid p_1) - \mu[(p_2 - p_1)d]| \leq \frac{2d}{L} \max_{x \in [d, d]} |\mu(x)| + \frac{2d}{L} \max_{x \in [-2d, 2d]} |\mu(x)|,$$ 

\(^6\)Lemma 3 in the Online Appendix of Kőszegi and Rabin (2009) states a similar result, but for a different order of limits.
an expression not depending on $p_1, p_2$. The max terms are seen to be finite by applying extreme value theorem to the continuous $\mu$, so the RHS tends to 0 as $L \to \infty$. ☐