Dynamic Information Design with Diminishing Sensitivity Over News

Jetlir Duraj   Kevin He

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Introduction

- People sometimes willing to pay to change how they receive news over time, though info does not help decision-making
- Bob phones relative in foreign country once a week. Today, relative is quarantined due to possible virus exposure
  - If infected, relative will fall sick next week
  - Bob changes routine and calls relative daily to get multiple pieces of reassuring news
- This paper: Elated by good news, disappointed by bad news, **diminishingly sensitive (DS)** reactions to news
- Two congruent pieces of news in 2 periods have greater total impact than if all the news were received in 1 period
- Study implications for **info preference** and **dynamic communication**

Key Assumption: Diminishing Sensitivity

- Classical assumption of ref-dep preferences since Kahneman and Tversky (1979)
- Larger deviations carry smaller marginal effects
- Based on Weber's law and human perception of stimuli

But, almost all papers on ref-dep in past 40 years **assume away** DS for simplicity, instead use a two-part linear gain-loss utility

O'Donoghue and Sprenger (2018)'s survey of ref-dep models:

> *"Most applications of reference-dependent preferences focus entirely on loss aversion, and ignore the possibility of diminishing sensitivity. [...] The literature still needs to* ***develop a better sense of when diminishing sensitivity is important****."*

How Diminishing Sensitivity over News Matters

This project: DS leads to novel, testable predictions about info preference and info transmission

Predictions of some existing models:

- Classical preference: indifferent between all info structures
- Kőszegi and Rabin (2009): news utility without DS
  - optimal info structure: one-shot resolution
- Ely, Frankel, Kamenica (2015): utility from magnitude of belief change ("suspense and surprise")
  - cannot talk about "good" or "bad" news
  - worst info structure: one-shot resolution

Will discuss news utility with DS in three environments:

1. Choosing between gradual info vs. one-shot info
2. Informed sender with commitment power
3. Informed sender without commitment power

Model: Timing and Beliefs

Discrete time $t \in \{0, 1, ..., T\}$

Exogenous state $\theta \in \Theta = \{A, B\}$, prior $\mathbb{P}[\theta = A] = \pi_0 \in (0, 1)$

- Agent consumes in period T and gets consumption utility $v(\theta)$
- No consumption in other periods

In each period $t = 1, ..., T$, agent observes some info about state

- Forms Bayesian posterior prob $\pi_t \in [0, 1]$ about $\{\theta = A\}$
- Non-instrumental info: consumption unaffected by actions
- Perfectly learns θ at moment of consumption $t = T$, $\pi_T \in \{0, 1\}$

Each period, derives utility from **consumption news** — recent change in belief about expected future consumption utility

Model: News Utility

At end of period t, gets

$$\mu\left[\ (\pi_t v(A) + (1-\pi_t)v(B)) - (\pi_{t-1}v(A) + (1-\pi_{t-1})v(B))\ \right]$$

(Paper has results about environments with more states and more general news-utility functions $N : \Delta(\Theta) \times \Delta(\Theta) \to \mathbb{R}$)

Objective: expected total news utility $\mathbb{E}[\sum_{t=1}^{T} \mu(\pi_t - \pi_{t-1})]$

- Time neutral preference

Maintained assumptions on μ

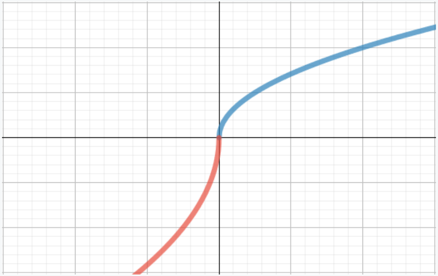
- $\mu(0) = 0$, μ strictly increasing
- **DS**: $\mu''(x) < 0$ for $x > 0$, $\mu''(x) > 0$ for $x < 0$
- **Weak loss aversion** (**LA**): $\mu(x) \leq -\mu(-x)$ for $x > 0$
  - includes μ symmetric

For $\tilde{\mu} : \mathbb{R}_+ \to \mathbb{R}_+$ strictly increasing and concave, can define a λ-**scaled family** of news-utility functions $(\mu_\lambda)_{\lambda \geq 1}$ where $\mu_\lambda(x) = \tilde{\mu}(x)$ for $x \geq 0$, $\mu_\lambda(x) = -\lambda\tilde{\mu}(-x)$ for $x < 0$.

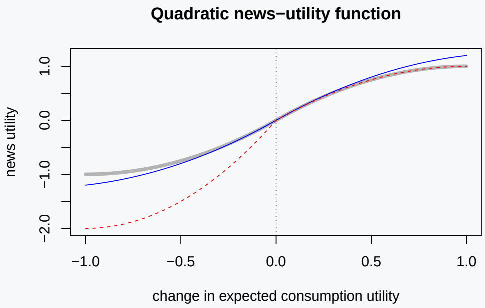
Example: Square-Roots $\lambda-$Scaled Family

$$\mu(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ -\lambda\sqrt{-x} & x < 0 \end{cases}$$

with $\lambda \geq 1$. Illustration for $\lambda = 2$:



Example: Quadratic μ



$$\mu(x) = \begin{cases} \alpha_p x - \beta_p x^2 & x \geq 0 \\ \alpha_n x + \beta_n x^2 & x < 0 \end{cases}$$

	α_p	α_n	β_p	β_n
Grey	2	2	1	1
Blue	2	2	0.8	0.8
Red	2	4	1	2

- $\alpha_n - \alpha_p$: loss aversion near 0; β_p, β_n: curvature, i.e. DS
- Add parameter restrictions to ensure monotonic, weak LA

Gradual Info vs. One-Shot Info

A sequence of signal realizations determine the state gradually

- In each $t = 1, 2, \ldots, T$, random variable $X_t \in \{0, 1\}$ realizes, $\mathbb{P}[X_t = 1] = q_t \in (0, 1)$, X_t's independent
- If $X_t = 1$ for **all** t, state is A. Else, state is B

At $t = 0$, agent commits to either observing realizations $(X_t)_{t=1}^{T}$ in real time, or only learning the final state in period T

- **Example**: debate between political candidates A and B, A loses as soon as she makes a "gaffe," but wins if no gaffe
  - Agent chooses between watching the debate or reading its outcome tomorrow
  - $v(A) \gtrless v(B)$ depends on agent's preference over candidates
- Consider a λ-scaled family of news utility functions (μ_λ)

Endogenous Diversity of Information Preferences

Proposition

For any $\lambda \geq 1$, an agent who prefers state B will choose one-shot information. There exists some $\bar{\lambda} > 1$ so that for any $1 \leq \lambda \leq \bar{\lambda}$, an agent who prefers state A will choose gradual information.

$v(A) > v(B)$: watch events unfold to celebrate small victories

$v(B) > v(A)$: only learn final state to avoid piecemeal bad news

- Agents with opposite **consumption** preferences over A, B exhibit opposite **info** preferences
- Single agent with stable news-utility function makes **different info choices in different environments**

Proposition

These models predict agent does not change his info choice when $v(A) \gtrless v(B)$ changes: (1) News utility with two-part linear μ; (2) Anticipatory utility; (3) Suspense and surprise.

Application 1: Media Competition

- Suppose realization of some state A depends on a series of smaller events
- Some news outlets may cover these small events in detail
- Others only report final outcome
- Viewers sort between news sources based on $v(A) \gtrless v(B)$

Application 2: Game Shows

- 5 rounds, 1 contestant who will win either \$100,000 or nothing
- Empathetic audience gets news utility $\mu(\pi_t - \pi_{t-1})$ at the end of round t, $\pi_t =$ Bayesian prob. of contestant winning \$100k
- Format 1: "sudden death" — 5 easy rounds each with 87% winning probability, win \$100k if win **all** 5 rounds
- Format 2: "repêchage" — 5 hard rounds each with 13% winning probability, win \$100k by winning **any** round
- Proposition ⇒ audience whose LA is not too high will watch Format 1, but no one will watch Format 2
- Vast majority of game shows resemble Format 1
  - Even though two formats have same prob. of winning and same suspense and surprise utilities

Informed Sender with Commitment Power

Normalize $v(A) = 1, v(B) = 0$

A benevolent sender knows the state and maximizes agent's (receiver's) welfare

θ is sender's private info, observed at start of $t = 1$

At $t = 0$, sender publicly commits to an **info structure** (M, σ)

- M – finite message space
- $h^t \in (M)^t$ – public history of messages at end of period t
- $\sigma = (\sigma_t)_{t=1}^{T-1}$, $\sigma_t(\cdot \mid \theta, h^{t-1}) \in \Delta(M)$ – how sender mixes over messages in period t, depending on state and public history
- Receiver knows (M, σ), forms Bayesian posterior belief π_t after history h^t at end of t
- **Remark**: equivalent to a single-agent setting where receiver freely chooses any info environment for himself

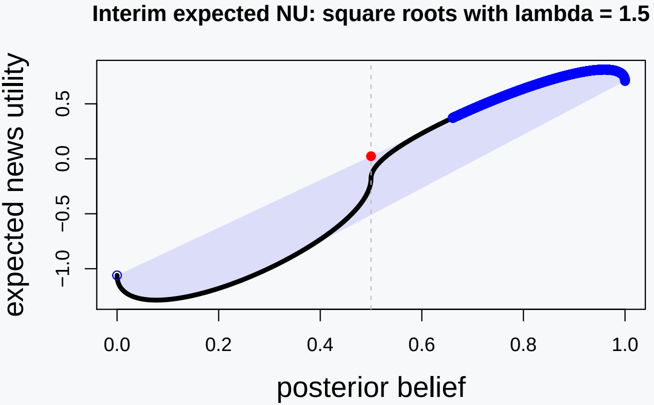
An Example

As an example, consider an environment with

- Time horizon $T = 5$
- Prior $\pi_0 = 0.5$
- Square-roots news utility with loss aversion:
  - $\mu(x) = \sqrt{x}$ for $x \geq 0$
  - $\mu(x) = -1.5\sqrt{-x}$ for $x < 0$
- Suppose receiver enters last period of communication with belief 0.5
- Sender communicates today, tomorrow true state revealed
- **Interim expected NU** if update posterior to p:

$$U_{T-1}(p \mid 0.5) := \underbrace{\mu(p - 0.5)}_{\text{NU today}} + \underbrace{p\mu(1-p) + (1-p)\mu(-p)}_{\text{expected NU tomorrow (by MG property)}}$$

Solving the Auxiliary Problem



- U **non-monotonic** due to μ DS
  - If μ two-part linear, then U monotonic
- Denote best expected NU following $\pi_{T-2} = 0.5$ by $U^*_{T-1}(0.5)$

Optimal Info Structure via Backward Induction

Once **value function** $U^*_{t+1}(\pi_t)$ defined for all $\pi_t \in \Delta(\Theta)$, let

$$U_t(\pi_t \mid \bar{\pi}_{t-1}) := \underbrace{\mu(\pi_t - \bar{\pi}_{t-1})}_{\text{NU in } t} + \underbrace{U^*_{t+1}(\pi_t)}_{\text{expected NU } \geq t+1}$$

$$U^*_t(\bar{\pi}_{t-1}) := [\text{cav} U_t(\cdot \mid \bar{\pi}_{t-1})]_{\pi_t = \bar{\pi}_{t-1}}$$

for each $\bar{\pi}_{t-1} \in \Delta(\Theta)$ — the value function one period back

Proposition

The optimal news utility is $U^*_1(\pi_0)$.

In general, hard to solve optimal info structure in closed form (so many results will focus on qualitative features of receiver's preference over info structures). But, can explicitly solve for quadratic news utility (later).

Optimal Info Structure in the Example

- Two messages, $\{a, b\}$
- If $\theta = B$, send b in a **random** period $\tilde{t}$
  - In periods before $\tilde{t}$, send a
- If $\theta = A$, send a every period
- b = conclusive bad news, a = partial good news
- Optimal info structure features **gradual good news, one-shot bad news**

When $\theta = A$, belief grows at **increasing rates** across periods:

period	0	1	2	3	4	5
belief in $\theta = A$	0.500	0.556	0.626	0.715	0.834	1.000
change in belief		0.056	0.070	0.089	0.119	0.166

Uneven growth because of cross-state constraints

When Is One-Shot Resolution Suboptimal?

If μ two-part linear with loss aversion,

$$\mu(x) = \begin{cases} x & x \geq 0 \\ \lambda x & x < 0 \end{cases}$$

for $\lambda \geq 1$, then one-shot resolution is optimal.

Proposition

One-shot resolution is strictly suboptimal if

$$\underbrace{\mu(1-\pi_0) - \mu(-\pi_0)}_{\text{always positive}} + \underbrace{\mu'(0^+) - \mu'(1-\pi_0)}_{\text{positive by DS}} + \underbrace{\mu(-1)}_{\text{DS vs. LA}} > 0.$$

- Need "enough DS" (in pos. or neg. region) relative to LA

When Is One-Shot Resolution Suboptimal?

Example: quadratic news utility with $\alpha_n - \alpha_p < \beta_n + \beta_p$.

- LHS is size of kink at 0, i.e., LA near 0
- RHS relates to $\mu^{''}(x)$ in negative and positive regions, i.e. DS

Example: any news utility function from the square-roots λ-scaled family (or more generally, any power function)

- “infinite” DS near 0, as $\mu^{''}(0^{+}) = -\infty$ and $\mu^{''}(0^{-}) = \infty$

Gradual Good News and Gradual Bad News

In fact, some info structure with “strictly gradual good news, one-shot bad news” strictly better than one-shot resolution if condition holds.

Definition

(M, σ) features **gradual good news, one-shot bad news** if:

- $\mathbb{P}_{(M,\sigma)}[\pi_t \geq \pi_{t-1}$ for all $1 \leq t \leq T \mid A] = 1$
- $\mathbb{P}_{(M,\sigma)}[\pi_t < \pi_{t-1}$ for at most one $1 \leq t \leq T \mid B] = 1$

There is **strictly gradual good news** if

- $\mathbb{P}_{(M,\sigma)}[\pi_t > \pi_{t-1}, \pi_{t'} > \pi_{t'-1}$ for two distinct $t \neq t' \mid A] > 0$

Similarly define **gradual bad news**.

Gradual Good News and Gradual Bad News

What about info structures with the "opposite skewness"?

Proposition

Under DS and weak LA, any info structure featuring strictly gradual bad news, one-shot good news is ***strictly worse*** *than one-shot resolution.*

- Note this also applies to symmetric μ
- Interpretation: DS implies a preference over skewness
- By contrast, no info structure is worse than one-shot resolution under suspense and surprise

Optimal Info Structure with Quadratic News Utility

Proposition

For $T = 2$ and quadratic news utility satisfying $\alpha_n - \alpha_p < \beta_n + \beta_p$, the optimal info structure induces a belief of either 0 or p_H in period $t = 1$, where p_H solves:

$$\pi_0(\alpha_n - \alpha_p) - (\beta_p + \beta_n)\pi_0^2 = p_H^2(\alpha_n - \alpha_p + \beta_n + \beta_p) - p_H^3(2\beta_p + 2\beta_n).$$

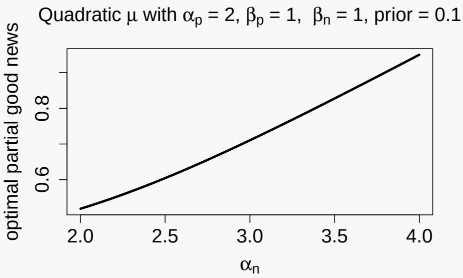
Find optimal info structure by solving a cubic equation

Two comparative statics emerge. Let $c := \frac{\alpha_n - \alpha_p}{\beta_n + \beta_p}$

- $\frac{dp_H}{dc} > 0$, so optimal partial good news increases in LA around reference point, decreases in DS
- $\frac{dp_H}{d\pi_0} < 0$ when $\pi_0 < \frac{1}{2}c$, but $\frac{dp_H}{d\pi_0} > 0$ when $\pi_0 > \frac{1}{2}c$

LA vs. DS in Shaping the Optimal Info Structure

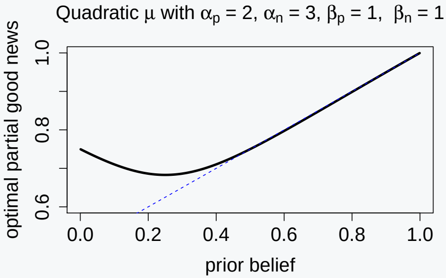
Optimal p_H increasing in loss aversion around ref. point (i.e., α_n)



- Sometimes, partial good news in bad state. Positive news utility today, more disappointment tomorrow. Sender picks:
  1. larger magnitude false hope, lower probability; or
  2. smaller magnitude false hope, higher probability
- Higher LA around reference point makes "small gain then small loss" more costly $\Rightarrow$ (1) is better
- Higher DS narrows the utility gap between the positive components of (1) and (2) $\Rightarrow$ (2) is better

Non-Monotonicity of p_H in Prior

When there's LA, p_H decreases then increases with the prior belief



Opposing incentives in maximizing utility given $\omega = A$ and $\omega = B$

- $\omega = A$: want $p_H = (\pi_0 + 1)/2$ to split news evenly (by DS)
- $\omega = B$: LA distorts towards bigger (and rarer) good news

$\{\omega = B\}$ welfare matters more for π_0 small, so $p_H \gg (\pi_0 + 1)/2$

$\{\omega = A\}$ welfare more important as π_0 grows, so p_H converges to $(\pi_0 + 1)/2$

Cheap Talk Model when Sender Lacks Commitment

- Let finite message space M be fixed with $|M| \geq 2$
- Sender **cannot commit** to strategy $\sigma_t(\cdot \mid \theta, h^{t-1})$
- Solve for **perfect-Bayesian eqm** under a belief refinement:
  - Once receiver updates belief to 0 or 1, belief stays constant through end of period $T-1$
  - In period T, belief $=$ full confidence in true state
- The babbling equilibrium always exists
- No other equilibrium payoff if μ exhibits DS but not LA

Proposition

When μ is symmetric around the origin and $\mu''(x) < 0$ for all $x > 0$, the babbling equilibrium is unique up to payoffs for any T.

Receiver's DS leads to a credibility problem for the sender.

Diminishing Sensitivity and the Credibility Problem

Consider period $T-1$

- When $\theta = B$, sender strictly prefers to lie and send a positive message intended for $\theta = A$
  - A trade-off for lying: positive news utility in period $T-1$ versus greater disappointment in $t = T$ due to false hope
  - **Due to DS**, trade-off favors lying
  - marginal utility of positive news today $>$ marginal disutility of heightened future disappointment
- So in eqm, sender cannot convey good news in **any** state
  - Else, tempted to mimic good-news message when $\theta = B$, destroying the news' credibility

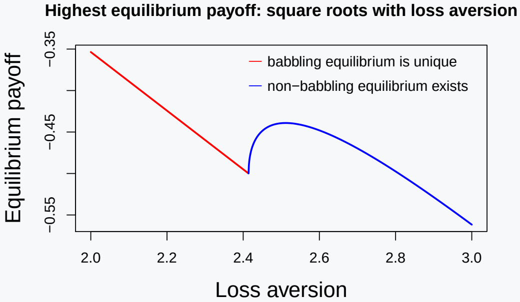
Sender must babble in period $T-1$

Now apply backward induction

This argument requires μ symmetric (i.e. $\lambda = 1$). Non-babbling equilibria can exist when LA high.

Loss Aversion and Receiver Welfare

Suppose $\mu(x) = \sqrt{x}$ for $x \geq 0$, $\mu(x) = -\lambda\sqrt{-x}$ for $x < 0$, $T = 2$



- Red: higher λ linearly decreases welfare (non-strategic effect)
- Blue: non-babbling eqm exists, λ additionally changes the eqm allocation of good news to different periods (eqm effect)
- For intermediate λ, eqm effect improves welfare by improving "consumption smoothing" of good news

GGN Equilibria

Definition

An equilibrium features deterministic gradual good news (**GGN equilibrium**) if there are constants $p_0 \leq p_1 \leq \ldots \leq p_T$ with $p_0 = \pi_0, p_T = 1$ s.t. $\pi_t = p_t$ in every t when $\theta = A$. The number of distinct beliefs in $(\pi_0, 1)$ is the number of **intermediate beliefs**.

Can characterize set of GGN equilibria using a relationship between successive intermediate beliefs.

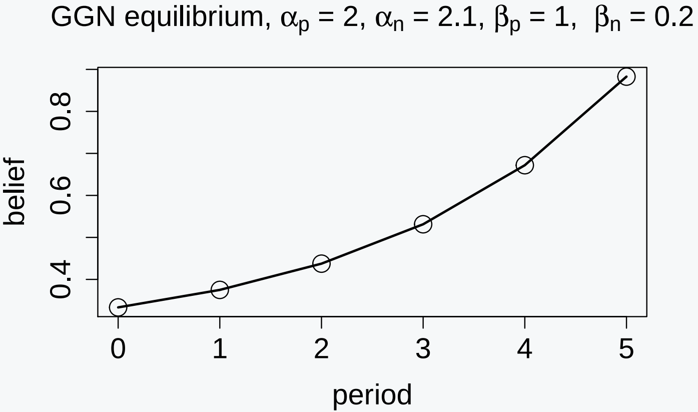
Proposition

There exists a GGN equilibrium with intermediate beliefs $q^{(1)} < \ldots < q^{(J)}$ if and only if $q^{(j)} \in P^*(q^{(j-1)})$ for all j (we put $q^{(0)} := \pi_0$), where

$$P^*(\pi) := \{\pi < x < 1 : \mu(x - \pi) + \mu(-x) = \mu(-\pi)\}.$$

GGN Equilibria for Quadratic μ

Example: For quadratic μ, $P^*(\pi) = \left\{\pi \cdot \frac{\beta_p+\beta_n}{\beta_p-\beta_n} - \frac{\alpha_n-\alpha_p}{\beta_p-\beta_n}\right\} \cap (\pi, 1)$.
GGN with multiple intermediate beliefs only exist for $\beta_n < \beta_p$.



Speed of Learning in GGN Equilibrium

Corollary

For μ quadratic or square roots, intermediate beliefs satisfy $q^{(j)} - q^{(j-1)} < q^{(j+1)} - q^{(j)}$ in every GGN equilibrium.

- Sender releases progressively larger pieces of good news
  - Intuition: when $\theta = B$, if sender indifferent between d amount of false hope and truth-telling at belief π_L, then strictly prefers d false hope over truth-telling at any $\pi_H > \pi_L$ due to DS
- Uniform bound on number of periods of informative communication, no matter how long the horizon T is
  - Bound is 5 for the quadratic example shown before
- More generally, "increasing rate of learning" holds whenever:
  - Sender indiff betw $\pi \to x \to 0$ and $\pi \to 0$ for at most one x
  - An ϵ amount of false hope better than truth-telling

Summary: Implications of Diminishing Sensitivity

DS leads to **more nuanced info preference**

- Agent's preference between gradual and one-shot **info** depends on **consumption** preferences over states
- Preference over the direction of news **skewness**
  - Some strictly gradual good news, one-shot bad news info structure strictly improves on one-shot resolution (under a sufficient condition)
  - Opposite skewness (i.e. gradual bad news) always worse than one-shot resolution

DS leads to **novel credibility problems** without commitment

- If low loss aversion, **babbling** is the unique equilibrium
- Receiver's equilibrium welfare **non-monotonic** in loss aversion
- In GGN equilibria, for quadratic and square-roots μ, receiver gets **progressively larger** pieces of good news over time

Thank you!

More Related Literature

- News utility without DS: Kőszegi and Rabin (2009), Pagel (2016, 2017, 2018), Duraj (2018 WP)
- Choosing beliefs without constraints (rather than info structure): Brunnermeier and Parker (2005), Macera (2014)
- Axioms of belief-based utility for $T = 2$: Dillenberger and Raymond (2020 WP)
- Bowman, Minehart, and Rabin (1999): ref-dep consumption model **with DS**
  - But, ref points are past habits, not rational expectations
  - All consumption loss in period 1, spread out consumption gains
  - Analogous strategy infeasible in sender's info design problem
- Lipnowski and Mathevet (2018) discuss an application to news utility without DS
  - We study information preference implications of DS, look at dynamics, relax commitment
- Li and Norman (2021), Wu (2018 WP)
  - Group of senders move sequentially to persuade one receiver
  - Receiver acts once after all senders, only final belief matters
  - Our problem: final belief of receiver exogenous, but **stochastic process of interim beliefs** is the object of design